A Criterion for Holism in Quantum Mechanics

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∞ Motivation ∞

- The question whether or not quantum mechanics (QM) gives rise to some form of **holism** has generally been answered to the positive.

  Why? What is it that makes quantum mechanics a holistic theory (if so), and other physical theories not (if so).

- I propose an **operational criterion** to decide whether or not a physical theory is **holistic**. That is, to decide from the formalism, the operations it allows for (things we can do) and from some property assignment rules whether or not the theory thus interpreted is holistic.

- I want to **contrast** my approach to the standard approach of holism in terms of **supervenience**, by showing that the latter approach should be rejected because it is too limited to satisfactory address the issue.

∞ Outline ∞

1. **Standard approach to Holism and how I address Holism**

2. **Operational Criterion for Holism**

3. **Orthodox Quantum Mechanics**

   - Criterion for Holism in the Quantum Formalism

   - Orthodox QM is Holistic without Entanglement

4. **Conclusion**

∞ The standard approach of Holism ∞

The supervenience stance on Holism (within a theoretical framework):

A theory is holistic iff some of the properties of the whole do not supervene on **(i)** the properties of the parts and **(ii)** on the mutual interactions that can occur according to the theory.

Paradigmatic example of a Holistic theory would then be Orthodox Quantum Mechanics.

- The singlet and the triplet state

$$\left|\,\phi^+\right\rangle = |\,00\rangle_z + |\,11\rangle_z\,, \qquad \left|\,\psi^-\right\rangle = |\,01\rangle_z - |\,10\rangle_z\,,$$

have completely **different** spin properties (complete anti-correlation and complete correlation). However in both cases the individual systems have the **same** reduced state and no spin property at all.

- Thus to a difference in global properties does not correspond a difference in the local properties of the subsystems. Therefore there is no supervenience and the theory is Holistic.

- I want to approach this example from a different point of view, and show that the supervenience stance on holism is **not the full story** because it is too idealised.

I look at what non-holistic physical processes according to the theory **can actually be performed** instead of solely considering state descriptions. Thus I take more of **an operational stance**.

- It then is possible to determine, using only local means and classical communication, whether or not one is dealing with the singlet or with the triplet state.

How? Measure on each subsystem the spin in the z-direction. Then compare the results using classical communication. If the result have the same parity the state was the triplet. If the parity is not the same the state was the singlet.

- Thus using local means and classical communication the two Bell states can be distinghuised and the two different global properties can be obtained after all. **There is no indication of holism in this case.**

∞ **How to adress holism in a theory?** ∞

- Some form of **property assignment** is necessary.
- Some theories give a **natural** property assignment.

  Others do not and then **an interpretation** is necessary.

Example: Quantum Mechanics

- Bohmian Mechanics: Property assignment as in classical physics.
- Orthodoxy: Property assignment in terms of eigenstate-eigenvalue link.

1 Focus on the properties assigned to a composite system and its parts. These are the **global** and **local** properties respectively.

2 What sort of theories are **candidates** for holism?

Theories that contain global properties which cannot be determined from the local properties assigned to the sub-systems and from their mutual interactions.

3 Consider physical (non-holistic) **constraints** on the determination (measurement) of the property assignments.

Proposed Method is to take an operational stance:

- Study the physical **realizability** of measuring global properties that occur in a theory.
- Take as a **constraint** that one only uses local operations and classical communication (LOCC).

Guiding Idea: Local operations and classical communication provide us with only the local properties of the parts while taking into account the possible mutual interactions.

∞ Operational Criterion for Holism ∞

A physical theory with a property assignment rule is holistic iff some determination (measurement) of the global property assignments cannot be implemented by LOCC.

- Criterion works for all theories with a property assignment rule and a specification of what LOCC is in the theory.

- If a theory is not holistic in the supervenience approach it is also not in this approach, but not the other way around.

Operational Approach $\Longrightarrow$ **Supervenience Approach**

- If properties of the whole supervene on properties of the parts then measuring these properties allows for determination of the properties of the whole.

- However, if global properties of the whole do not supervene on properties of the parts, it could be the case that the global properties may be obtainable using LOCC.

∞ **Orthodox Quantum Mechanics** ∞

- Property assignment rule via **eigenvalue-eigenstate link**:

A physical system has the property that quantity $\mathfrak{B}$ has a fixed value iff its state is an eigenstate of the operator $\hat{B}$ corresponding to $\mathfrak{B}$. This value is the eigenvalue belonging to the corresponding eigenstate.

- All measurements are ideal von Neumann measurements.

∞ Criterion for Holism in the Quantum Formalism ∞

Orthodox Quantum theory is holistic iff some of its global property assignments can not be determinded by local quantum operations and classical communication (LOCC).

Idea: LOCC provides us with properties of the parts, and it also takes into account the mutual interactions that may occur.

- **Then what is LOCC?**

A general quantum process takes a state ρ^A of a system A on $\mathcal{H}_1$ to a state σ^A on $\mathcal{H}_2$:

$$\rho^A \to \sigma^A = \mathcal{S}(\rho^A), \qquad \rho^A \in \mathcal{H}_1, \;\; \mathcal{S}(\rho^A) \in \mathcal{H}_2$$

$\mathcal{S} : \mathcal{H}_1 \to \mathcal{H}_2$ is a **completely positive trace-nonincreasing map**, i.e. an operator $\mathcal{S}$ acting linearly on Hermitian matrices such that $\mathbb{1} \otimes \mathcal{S}$ takes density operators to density operators.

These maps are also called **quantum operations**.

- **LOCC operations** is the class of local operations plus two-way classical communication. It consists of composition of the following two elementary operations

$$\mathcal{S}^A \otimes \mathbb{1},$$
$$\mathbb{1} \otimes \mathcal{S}^B.$$

with $\mathcal{S}^A$ and $\mathcal{S}^B$ local quantum operations.

Example: A communicates her result α to B, after which B performs his measurement: $\mathcal{S}^{AB}(\rho) = (\mathbb{1} \otimes \mathcal{S}^B_\alpha) \circ (\mathcal{S}^A \otimes \mathbb{1})\rho$.

∞ Orthodox QM is holistic not needing entanglement ∞

Suppose we have a physical quantity $\mathfrak{R}$ with a corresponding operator $\hat{R}$ that has a set of nine eigenstates, $|\,\psi_1\rangle$ to $|\,\psi_9\rangle$, with eigenvalues 1 to 9.

The **property assignment** we consider is: If the system is in an eigenstate $|\,\psi_i\rangle$ then it has the property that quantity $\mathfrak{R}$ has the fixed value i.

Suppose $\hat{R}$ works on $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ (each three dimensions) and has the following complete orthonormal set of eigenstates:

$$
\begin{aligned}
|\,\psi_1\rangle &= |\,1\rangle \otimes |\,1\rangle \\
|\,\psi_2\rangle &= |\,0\rangle \otimes |\,0+1\rangle \\
|\,\psi_3\rangle &= |\,0\rangle \otimes |\,0-1\rangle \\
|\,\psi_4\rangle &= |\,2\rangle \otimes |\,1+2\rangle \\
|\,\psi_5\rangle &= |\,2\rangle \otimes |\,1-2\rangle \\
|\,\psi_6\rangle &= |\,1+2\rangle \otimes |\,0\rangle \\
|\,\psi_7\rangle &= |\,1-2\rangle \otimes |\,0\rangle \\
|\,\psi_8\rangle &= |\,0+1\rangle \otimes |\,2\rangle \\
|\,\psi_9\rangle &= |\,0-1\rangle \otimes |\,2\rangle\,,
\end{aligned}
$$

with $|\,0+1\rangle = \frac{1}{\sqrt{2}}(|\,0\rangle + |\,1\rangle)$, etc.

We want to determine if the composite system has the property that the value of the observable $\mathfrak{R}$ is one of the numbers 1 to 9, using only LOCC operations performed by two observers A and B that each have one of the individual subsystems.

- This amounts to determining which eigenstate A and B have or project on during the measurement.

If A and B project on eigenstate $|\,\psi_i\rangle$ then to the measurement outcome i there is associated a quantum operation
$\mathcal{S}_i : \rho \to \frac{\mathcal{S}_i(\rho)}{\mathrm{Tr}[\mathcal{S}_i(\rho)]}$, with projection operators $S_i = |\,i\rangle_A\,|\,i\rangle_B\,\langle\,\psi_i|$.

The state $|\,i\rangle_A$ denotes the classical record of the outcome of the measurement that A writes down, and similarly for $|\,i\rangle_B$.

These classical records can be considered to be **local** properties of the subsystems of A and B.

Implementing the quantum operation $\mathcal{S}(\rho) = \sum_i \mathcal{S}_i \rho \mathcal{S}_i^\dagger$ amounts to determining the **global** property assignment given by $\hat{R}$.

- **This cannot be done using LOCC.**

(Bennett and co-workers. PRA 59, 1070 (1999)).

- **Sketch of the proof**:

If A or B perform projective measurements in any of their operation and communication rounds then the distinghuishability of the states is spoiled.

Spoiling occurs in **any** local basis.

The ensemble of states as seen by A or by B alone is non-orthogonal.

Conclusion: A physical quantity, whose corresponding operator has only product eigenstates, has a property assignment using the eigenvalue-eigenstate link that is not implementable using LOCC.

Thus quantum mechanics is holistic without the need of entanglement.

∞ Conclusion ∞

- I sketched an operational criterion for holism that determines whether or not a theory with a property assignment rule is holistic.

- The supervenience approach is shown to be of limited use because it neglects operational criteria that theories allow for.

  'We can do more or less than (quantum) states at first seem to tell us.'

- Orthodox quantum mechanics is shown to be holistic, without the need of entanglement.

- This is an example of the idea that we might get fundamental new insights from investigating what we **can** and **can not do** quantum mechanically.

This I call the **thermodynamical analogy**, because from investigating what we can and can not do thermodynamically we obtain the second law of thermodynamics.