# A Criterion for Holism in Quantum Mechanics

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# ∞ Motivation ∞

- The question whether or not quantum mechanics (QM) gives rise to some form of holism has generally been answered to the positive.
  Why? What is it that makes quantum mechanics a holistic theory (if so), and other physical theories not (if so).

- I propose an operational criterion to decide whether or not a physical theory is holistic. That is, to decide from the formalism, the operations it allows for (things we can do) and from some property assignment rules whether or not the theory thus interpreted is holistic.

- I want to contrast my approach to the standard approach of holism in terms of supervenience, by showing that the latter approach needs to be rejected since it is too idealistic and abstract.

# ∞ Outline ∞

1. **Standard approach to Holism and how I address Holism**

2. **Operational Criterion for Holism**

3. **Examples**

   - Newtonian Gravity and Bohmian Mechanics are not Holistic

4. **Orthodox Quantum Mechanics**

   - Criterion for Holism in the Quantum Formalism
   - Orthodox QM is Holistic without Entanglement

5. **Conclusion**

## $\infty$ The standard approach of Holism $\infty$

**The supervenience stance on Holism** (within a theoretical framework):

The properties of the whole do not supervene on the properties of the parts and on their mutual interactions that can occur according to the theory.

**Paradigmatic example of a Holistic theory** would then be Orthodox Quantum Mechanics.

- The singlet $|\,\psi^-\rangle = |\,01\rangle - |\,10\rangle$, and the triplet state $|\,\phi^+\rangle = |\,00\rangle + |\,11\rangle$, have completely different spin properties (complete anti-correlation and complete correlation). However in both cases the individual systems have the same reduced state and no spin property at all.

- Thus to a difference in global properties does not correspond a difference in the properties of the subsystems. Therefore there is no supervenience and the theory is Holistic.

• I want to approach this example from a different point of view, and show that the supervenience stance on Holism needs to be rejected because it is too idealised and abstract.

I look at what non-holistic physical processes according to the theory can actually be performed. Thus I take more of an operational stance.

• It then is possible to determine, using only local means and classical communication, whether or not one is dealing with the singlet or with the triplet state.

How? Measure on each subsystem the spin in the $z$-direction. Then compare the results using classical communication. If the result have the same parity the state was the triplet. If the parity is not the same the state was the singlet.

$$\begin{aligned} \left|\phi^{+}\right\rangle &= \left|00\right\rangle_z + \left|11\right\rangle_z \\ \left|\psi^{-}\right\rangle &= \left|01\right\rangle_z - \left|10\right\rangle_z \end{aligned}$$

• Thus using local means and classical communication the two Bell states can be distinghuised and the two different global properties can be obtained after all. There is no indication of holism.

## ∞ How to adress holism in a theory? ∞

- Some form of property assignment is necessary.

- Some theories give a natural property assignment.

  Others do not and then an interpretation is necessary.

  **Example:** Quantum Mechanics

  - Bohmian Mechanics: Property assignment as in classical physics.

  - Orthodoxy: Property assignment in terms of eigenstate-eigenvalue link.

**1** Focus on the properties assigned to a composite system and its parts. These are the global and local properties respectively.

**2** What sort of theories are candidates for holism?

Theories that contain global properties which cannot be determined from the properties assigned to the sub-systems and from their mutual interactions.

**3** Consider physical (non-holistic) constraints on the determination (measurement) of the property assignments.

**Proposed Method is to take an operational stance:**

- Study the physical realizability of measuring global properties that occur in a theory.
- Take as a constraint that one only uses local operations and classical communication (LOCC).

**Guiding Idea:** Local operations and classical communication provide us with only the local properties of the parts, and further it also takes into account the mutual interactions.

## $\infty$ **Operational Criterion for Holism** $\infty$

A physical theory with a property assignment rule is holistic iff some determination (measurement) of the global property assignments cannot be implemented by LOCC.

- Criterion works for all theories with a property assignment rule and a specification of what LOCC is in the theory.

- If a theory is not holistic in the supervenience approach it is also not in this approach, but not the other way around.

- If properties of the whole supervene on properties of the parts then measuring these properties will allow one to determine the properties of the whole.

- However if global properties of the whole do not supervene on properties of the parts, it could very well be the case that the global properties may be obtainable using LOCC, rendering the theory not holistic.

$$\text{Operational Approach} \quad \Longrightarrow \quad \text{Supervenience Approach}$$

**In the rest of the talk**, I will study the LOCC implementability of observables that give global property assignments to composite systems. I try to show the following:

**1)** Orthodox QM is holistic, despite the feature of entanglement.

**2)** Newtonian Gravity and Bohmian Mechanics are not holistic, despite their non-local action at a distance.

## $\infty$ Example: Newtonian Gravity and Bohmian Mechanics $\infty$

**Natural property assignment rule:**

- Physical quantity $A$ is represented by a function $f_A : \Omega \to \mathbb{R}$ such that $f_A(x)$ is the value $A$ possesses when the state is $x$.

- To the property that the value of $A$ lies in $\Delta$ there is associated a subset

$$\Omega_{A\in\Delta} = f_A^{-1}\{\Delta\} = \{x \in \Omega | f_A(x) \in \Delta\},$$

of states in $\Omega$ for which the proposition that the system has this property is true.

- Thus propositions are associated with subsets of the space of states $\Omega$. Then the logical structure of the propositions about the physical properties of the system is identified with the **Boolean algebra structure** on the subsets of the space of states $\Omega$.

- **Composite system:** System properties can be derived from subsystem properties using LOCC.

To the property that the value of $B$ lies in $\Delta$ there is associated a subset, for which the proposition that the system has this property is true:

$$\begin{aligned}\Omega_{B\in\Delta} &= f_B^{-1}\{\Delta\} = \{x \in \Omega | f_B(x) \in \Delta\} \\ &= \{(x_1, x_2) \in \Omega | f_{A_1}(x_1, x_2) \in \Delta_1,\ f_{A_2}(x_2) \in \Delta_2\} \\ &= f_{A_1,A_2}\{\Delta_1, \Delta_2\},\end{aligned}$$

for some function $f_{A_1,A_2}$ and some subsystem physical quantities $A_1$ and $A_2$.

Examples of $f_{A_1,A_2}$:

$$\begin{aligned} x &= |\vec{x_1} - \vec{x_2}|, \\ \overrightarrow{F_1} &= -\nabla V(|\vec{x_1} - \vec{x_2}|), \\ \overrightarrow{F_1} &= -\nabla(V(\vec{q_1}, \vec{q_2}.t) + U(\vec{q_1}, \vec{q_2}, t)) = f(q_1^i, q_2^j, t),\end{aligned}$$

All three functions are highly non-local.

- **Conclusion:** All global property assignments can be determined from sub-system interactions and local properties, and the latter can be found by LOCC. Thus Newtonian Gravity and Bohmian Mechanics are not holistic.

## $\infty$ **Orthodox Quantum Mechanics** $\infty$

- Property assignment rule via eigenvalue-eigenstate link:

A physical system has the property that quantity $\mathfrak{B}$ has a fixed value iff its state is an eigenstate of the operator $\hat{B}$ corresponding to $\mathfrak{B}$. This value is the eigenvalue belonging to the corresponding eigenstate.

- All measurements are ideal von Neumann measurements.

## $\infty$ Criterion for Holism in the Quantum Formalism $\infty$

**Orthodox Quantum theory is holistic** iff some of its global property assignments can not be determinded by local quantum operations and classical communication (LOCC).

**Idea:** LOCC provides us with properties of the parts, also taking into account their mutual interactions.

- **Then what is LOCC?**

A general quantum process takes a state $\rho^A$ of a system $A$ on $\mathcal{H}_1$ to a state $\sigma^A$ on $\mathcal{H}_2$:

$$\rho^A \to \sigma^A = \mathcal{S}(\rho^A), \qquad \rho^A \in \mathcal{H}_1, \;\; \mathcal{S}(\rho^A) \in \mathcal{H}_2$$

$\mathcal{S} : \mathcal{H}_1 \to \mathcal{H}_2$ is a completely positive trace-nonincreasing map, i.e. an operator $\mathcal{S}$ acting linearly on Hermitian matrices such that $\mathbb{1} \otimes \mathcal{S}$ takes density operators to density operators.

These maps are also called *quantum operations*.

Each quantum operation has a Kraus representation:

$$\mathcal{S}(\rho) = \sum_\mu O_\mu \rho O_\mu^\dagger \quad \text{with} \quad \sum_\mu O_\mu O_\mu^\dagger \leq \mathbb{1}$$

- **LOCC operations** is the class of local operations plus two-way classical communication. It consists of composition of the following two elementary operations

$$\mathcal{S}^A \otimes \mathbb{1},$$
$$\mathbb{1} \otimes \mathcal{S}^B.$$

with $\mathcal{S}^A$ and $\mathcal{S}^B$ local quantum operations.

**Example:** $A$ communicates her result $\alpha$ to $B$, after which $B$ performs his measurement: $\mathcal{S}^{AB}(\rho) = (\mathbb{1} \otimes \mathcal{S}^B_\alpha) \circ (\mathcal{S}^A \otimes \mathbb{1})\rho$.

## $\infty$ **Orthodox QM is holistic not needing entanglement** $\infty$

Suppose we have a physical quantity $\mathfrak{R}$ with a corresponding operator $\hat{R}$ that has a set of nine eigenstates, $|\,\psi_1\rangle$ to $|\,\psi_9\rangle$, with eigenvalues 1 to 9.

The property assignment we consider is: If the system is in an eigenstate $|\,\psi_i\rangle$ then it has the property that quantity $\mathfrak{R}$ has the fixed value $i$.

Suppose $\hat{R}$ works on $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ (each three dimensions) and has the following complete orthonormal set of eigenstates:

$$\begin{aligned}
|\,\psi_1\rangle &= |\,1\rangle \otimes |\,1\rangle \\
|\,\psi_2\rangle &= |\,0\rangle \otimes |\,0+1\rangle \\
|\,\psi_3\rangle &= |\,0\rangle \otimes |\,0-1\rangle \\
|\,\psi_4\rangle &= |\,2\rangle \otimes |\,1+2\rangle \\
|\,\psi_5\rangle &= |\,2\rangle \otimes |\,1-2\rangle \\
|\,\psi_6\rangle &= |\,1+2\rangle \otimes |\,0\rangle \\
|\,\psi_7\rangle &= |\,1-2\rangle \otimes |\,0\rangle \\
|\,\psi_8\rangle &= |\,0+1\rangle \otimes |\,2\rangle \\
|\,\psi_9\rangle &= |\,0-1\rangle \otimes |\,2\rangle\,,
\end{aligned}$$

with $|\,0+1\rangle = \frac{1}{\sqrt{2}}(|\,0\rangle + |\,1\rangle)$, etc.

We want to determine if the composite system has the property that the value of the observable $\mathfrak{R}$ is one of the numbers 1 to 9, using only LOCC operations performed by two observers $A$ and $B$ that each have one of the individual subsystems.

- This amounts to determining which eigenstate $A$ and $B$ have or project on during the measurement.

If $A$ and $B$ project on eigenstate $|\,\psi_i\rangle$ then to the measurement outcome $i$ there is associated a quantum operation $\mathcal{S}_i : \rho \rightarrow \frac{\mathcal{S}_i(\rho)}{\mathrm{Tr}[\mathcal{S}_i(\rho)]}$ , with projection operators $S_i = |\,i\rangle_A\,|\,i\rangle_B\,\langle\,\psi_i|$.

The state $|\,i\rangle_A$ denotes the classical record of the outcome of the measurement that $A$ writes down, and similarly for $|\,i\rangle_B$.

These classical records can be considered to be properties of the subsystems of $A$ and $B$.

Implementing the quantum operation $\mathcal{S}(\rho) = \sum_i \mathcal{S}_i \rho \mathcal{S}_i^\dagger$ amounts to determining the property assignment given by $\hat{R}$.

- **This cannot be done using LOCC.**

  (Bennett and co-workers. PRA 59, 1070 (1999)).

- **Sketch of the proof**:

If $A$ or $B$ perform projective measurements in any of their operation and communication rounds then the distinghuishability of the states is spoiled.

Spoiling occurs in any local basis.

The ensemble of states as seen by $A$ or by $B$ alone is non-orthogonal.

**Conclusion:** A physical quantity, whose corresponding operator has only product eigenstates, has a property assignment using the eigenvalue-eigenstate link that is not implementable using LOCC.

Thus quantum mechanics is holistic without using entanglement.

# ∞ Conclusion ∞

- I sketched an operational criterion for holism that determines whether or not a theory with a property assignment rule is holistic.

- supervenience approach is rejected because it is too idealised, and can not incorporate operational citeria the theories allows for.

- Entanglement and non-locality do not lead to holism.

## The fruitfullness of looking at the implementation of observables $\propto$

"The commutativity of operators $\hat{A}$ and $\hat{B}$ is necessary and sufficient for the simultaneous measurability of [observables] $\mathfrak{A}$ and $\mathfrak{B}$."

*(J. Von Neumann, 'Mathematischen Grundlagen der Quantum-mechanik' p.121, Springer Verlag, Berlin, 1932)*

- $[\hat{A}, \hat{B}] = 0 \iff$ $\mathfrak{A}$ and $\mathfrak{B}$ are compatible.
- Identification of a mathematical concept (commutativity) with a physical concept (compatibility).

**Is commutativity sufficient for compatibility?**

## A possible counter example.

- Measure on a bi-partite spin-$\frac{1}{2}$ system, the operators $\hat{A}$ and $\hat{B}$ where $\hat{A} = \sigma_x \otimes \sigma_x$ and $\hat{B} = \sigma_z \otimes \sigma_z$.

$$[\hat{A}, \hat{B}] = [\sigma_x \otimes \sigma_x, \sigma_z \otimes \sigma_z] = 0, \quad \text{but} \quad [\sigma_x, \sigma_z] \neq 0.$$

- How to measure these global operators $\hat{A}$ and $\hat{B}$?

Since $\hat{A}$ and $\hat{B}$ commute they are a function of a third operator $\hat{C}$. It follows that both $\hat{A}$ and $\hat{B}$ have the same eigenbasis as $\hat{C}$, which is the complete Bell basis:

$$\left| \phi^{\pm} \right\rangle = |\, 00 \rangle \pm |\, 11 \rangle \,, \qquad \left| \psi^{\pm} \right\rangle = |\, 01 \rangle \pm |\, 10 \rangle \,.$$

**We must implement the Bell measurement operator.**

This can be done using local operations, classical communication and either shared randomness or shared entanglement.
Two parity bits are obtained that allows one to determine which Bell state is projected upon.

**Conclusion:**

**1):** The counterexample is rejected because simultaneous measurement of $\hat{A}$ and $\hat{B}$ can be implemented after all.

**2):** The Bell states and the property assignments they give rise to can be determined via LOCC. Thus the Bell states do not lead us to any holism, despite their entanglement.