

On the causal connection of the world. An extension of classical dynamics.

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(Submitted on 14 June 1922.)

[Translation¹ by M.P. Seevinck of *Über den Wirkungszusammenhang der Welt. Eine Erweiterung der klassischen Dynamik* by H. Tetrode, *Zeitschrift für Physik*, vol. 10, 317-328 (1922).]

It is well-known that electrodynamics can be used to calculate the force on an electron from the components of the electromagnetical field; in addition to this one only needs to include the velocity components of the electron and these are proportional to its charge. On the other hand, the Maxwell-Lorentz differential equations do not suffice to uniquely determine the field strengths, respectively the potentials, from the position and motion of the electron. It is true, however, that one can write down the general solutions for the potentials in a simple form; they consist of retarded integral solutions over the charge in some volume, and in addition [they consist of] of similar solutions over the potentials and their differentials on the surface of this volume. Instead of constructing retarded, or ‘delayed’ expressions, i.e., for a point source at a distance r for the state that existed there a time r/c ago, one can just as well construct ‘forwards rushing’ solutions, i.e., for a state which only after a time r/c will exist there. On mathematical grounds both results must be equal. Only in case the surface integrals are omitted they are in general distinguishable.

The fact that the motion of the electron – or at least changes of it – is determined by the field, yet not uniquely determined by it, is completely understandable when one regards the field as essential and primary, and by contrast the electron as particular singular modifications of it, determined only by deviations of the Maxwell-Lorentz equations in case of higher values of the field strength and potentials. There is then however no reason to suppose why the total field is determined by only one of its locations and there could very well exist fields that do not stem from the electron but have simply emerged from infinity. This particular “Theory of Matter” has been thoroughly given by Mie. However, it is subject to Hamilton’s principle and does not explain the quantum effects. These latter seem not at all compatible with the field idea.

¹This work was completed during a visit at the Center for Time, University of Sydney, Australia. Their hospitality and support is kindly acknowledged.

That is why I have chosen a different point of view and consider only that to be primary whose existence has been given to us experimentally by immediate experience, namely the –negative and positive– electrons, which, by the way, can even be counted and individually followed in their motion. I assume that every change of motion (acceleration) of an electron is determined by other electrons and consider the field as a mathematical construction that is suited to represent the interactions between the electrons in the limiting case where the quantum nature of the appearances can be neglected, but beyond which it leaves us in the lurge.

According to this principle we have to assume that the above mentioned general expression for the potentials give the correct value when we extend the volume integrals over all charges of the world, but leave out the surface integrals. Further, for the moment we would like to treat the positive and negative direction of time as equal. It then follows that we must take the arithmic mean of the retarded and advanced potentials.

We then obtain for the the four-potential Φ in well-known notation

$$\Phi = \frac{1}{2} \int [P]_t - \frac{r}{c} \frac{dS}{r} + \frac{1}{2} \int [P]_t + \frac{r}{c} \frac{dS}{r}, \quad (1)$$

where P is the four-current.

This can also be written as

$$\Phi = \int_{t'=-\infty}^{+\infty} c dt' \iiint_{-\infty}^{+\infty} \frac{f(\sigma^2)}{\sigma^2} P' dx' dy' dz', \quad (2)$$

where σ is the “world distance”

$$\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2 - c^2(t-t')^2} = \sqrt{r^2 - c^2(t-t')^2}$$

between every source point that has primed coordinates and one that has unprimed coordinates, and where the function f has to obey for each positive Δ different from zero the requirement

$$f(\pm\Delta) = 0 \quad \text{and} \quad \int_{-\Delta}^{+\Delta} \frac{f(x)}{x} dx = 1$$

(for negative Δ the integral is of course -1). From this it follows that only the infinitesimal surroundings of the “light cone” $\sigma = 0$ are relevant for the end result.

The contribution of a single electron of charge e' and velocity v to the potential is, because

$$e' = \int \frac{P'_l}{i} dS' \quad \text{and} \quad e' \frac{v'_x}{c} = \int P'_x dS' \quad \text{etc.,}$$

when we assume its dimensions to be very small:

$$\Phi_1 = \int_{-\infty}^{+\infty} \left(ie', e' \frac{v'}{c} \right) \frac{f(\sigma^2)}{\sigma^2} c dt' = e' \int \frac{f(\sigma^2)}{\sigma^2} d\varrho', \quad (3)$$

where $d\varrho'$ is the vectorial line element of the world line of this electron with components $d\varrho'_t = ic dt'$, $d\varrho'_x = dx'$, etc.

The integration is of course also in the second integral over the complete world line. When evaluating the first, one has to keep in mind that the spatial distance r which features in σ' is dependent on t' . One obtains:

$$\Phi_1 = \frac{1}{2} \left[\frac{\left(ie', e' \frac{v'}{c} \right)}{r \left(1 - \frac{v'_r}{c} \right)} \right]_{t - \frac{r}{c}} + \frac{1}{2} \left[\frac{\left(ie', e' \frac{v'}{c} \right)}{r \left(1 + \frac{v'_r}{c} \right)} \right]_{t + \frac{r}{c}}$$

corresponding to the well-known formula.

The Hamiltonian equation for the motion of an electron that has mass m_1 and charge e_1 in an electromagnetic field is known to be (with opposite sign compared to ordinary mechanics):

$$W_1 = m_1 \int \sqrt{-\Sigma d\varrho_1^2} - \frac{e_1}{c} \int (\Phi_1 \cdot d\varrho_1) = m_1 \int ds_1 - \frac{e_1}{c} \int (\Phi_1 \cdot d\varrho_1),$$

where $\varrho_{1,x}$ etc. is the spacetime coordinate of the electron, ds_1 is its world line element and the scalar multiplication is written in the usual way. The integrals range over the complete world line.

In case the field is caused by a second electron, then according to (3)

$$\Phi_1 = e_2 \int \frac{f(\sigma_{12}^2)}{\sigma_2^2} d\varrho_2,$$

because

$$W_1 = m_1 \int ds_1 - \frac{e_1 e_2}{c} \iint \frac{f(\sigma_{12}^2)}{\sigma_{12}^2} (d\varrho_1 \cdot d\varrho_2).$$

It is known that for an infinitesimal displacement of the world line of the first electron, which vanishes for $t = \pm\infty$, the requirement $\delta W_1 = 0$ gives its [the electron's] equations of motion in the field of the second [electron]; W_1 can be replaced by

$$W_{12} = m_1 \int ds_1 + m_2 \int ds_2 - \frac{e_1 e_2}{c} \iint \frac{f(\sigma_{12}^2)}{\sigma_{12}^2} (d\varrho_1 \cdot d\varrho_2). \quad (4)$$

because ϱ_2 is not varied. However, this expression is symmetrical in both indices and gives under displacement of ϱ_2 the equations of motion of the second electron in the field of the

first. Consequently is W_{12} the action for the motion of the two electrons under their mutual influence. This is no longer a Hamiltonian function because it cannot be reduced to a single time integral.

The expression

$$-\frac{e_1 e_2}{c} \iint \frac{f(\sigma_{12}^2)}{\sigma_{12}^2} (d\varrho_1 \cdot d\varrho_2)$$

is, up to the function F , the four-dimensional analogue of Neumanns electroynamical potential of twee electrical circuits

$$-i_1 i_2 \iint \frac{(dl_1 \cdot dl_2)}{r}.$$

The action for the entire world is:

$$W = \sum_i m_i \int ds_i - \sum_{\substack{ik \\ (i \neq k)}} \frac{e_i e_k}{c} \iint \frac{f(\sigma_{ik}^2)}{\sigma_{ik}^2} (d\varrho_i \cdot d\varrho_k), \quad (5)$$

where the first summation is extended over all electrons, the second over all electron pairs, where each pair is only to be counted once; in case we want to write them as a double summation, then a factor $\frac{1}{2}$ must be included; the terms with the same indices are then to be omitted because a double integral with such a term becomes infinite. The reason for this is that the finite electron diameter has not been taken into account; the single integrals $\int m_i ds_i$ replaces this.

Suppose one fixes the four-velocity of an electron to be $\frac{d\varrho_i}{ds_i} = v_i$ then one can also write

$$W = \sum_i m_i \int ds_i - \sum_{\substack{ik \\ (i \neq k)}} \frac{e_i e_k}{c} \iint \frac{f(\sigma_{ik}^2)}{\sigma_{ik}^2} (dv_i \cdot dv_k) ds_i ds_k. \quad (6)$$

We want to generalize this expression, which we have obtained from the classical equations of the electromagnetic field after reducing the mathematically possible solutions, immediately in such a way that it is to be expected that the quantum phenomena can be included and dealt with. Instead of the particular function f of σ_{ik}^2 we insert some, until now unknown, function of the four-velocity and the relative coordinates of both electrons. We can then write

$$W = \sum_i m_i \int ds_i + \sum_{\substack{ik \\ (i \neq k)}} \iint w_{ik} ds_i ds_k, \quad (7)$$

where $w_{ik} = w_{ki}$ is a scalar function of the three vectors v_i, v_k and $\varrho_i - \varrho_k = \sigma_{ik} = \sigma_{ki}$, that will be different for different electron pairs, i.e., that will be different for the three combinations negative-negative, positive-positive and negative-positive in case we only allow negative electrons and protons (hydrogen nuclei) as elementary particles.

The requirement $\delta W = 0$ gives, in the case where the displacements of the coordinates vanish for $t = \pm\infty$, for each index i the equation:

$$\frac{d}{ds_i} \left[m_i v_i - \sum_{k (k \neq i)} \int \left\{ \frac{\partial w_{ik}}{\partial v_i} + v_i \left(\frac{\partial w_{ik}}{\partial v_i} \cdot v_i \right) - v_i w_{ik} \right\} ds_k \right] = \frac{d\pi_i}{ds_i} = - \sum_{k (k \neq i)} \int \frac{\partial w_{ik}}{\partial v_i} ds_k, \quad (8)$$

where the expression in between the angled brackets is denoted by π_i . $\frac{d}{ds_i}$ stands for differentiation in the direction of the world line. The quantities with the index i are related to a definite world line point of the i -th electron, whereas the others with index k are integrated along its total world line.

We have for each electron four equations, corresponding to the four components of the vector equation (8). However, there must be some dependence amongst them, because the assignment of points of the world line that is varied to the world line that is not varied is within certain limits arbitrary and a change of this assignment makes no difference, because it maps the world line into itself. Actually, one obtains by scalar multiplication of (8) with v_i an identical equation when one realises that $(v_i \cdot v_i) = v_i^2 = -1$, and that

$$\frac{dw_{ik}}{ds_i} = \left(\frac{\partial w_{ik}}{\partial Q_i} \cdot v_i \right) + \left(\frac{\partial w_{ik}}{\partial v_i} \cdot \frac{dv_i}{ds_i} \right). \quad (9)$$

One observes further that for instance in case the constant quantity v_i^2 would explicitly appear in w_{ik} (which can be arbitrarily achieved) this would change nothing in the equations; for the elements that would appear through such a dependence in $\frac{\partial w_{ik}}{\partial v_i}$ and in $v_i \left(\frac{\partial w_{ik}}{\partial v_i} \cdot v_i \right)$ compensate each other.

The equations (8) entangle the motion of one electron to the motions and positions of all others at all possible times, even in the case where single time slices get more weight than others, analogous to classical theory where only certain time points are taken into account.

The functions w_{ik} must of course have certain convergence properties because otherwise the integrals in (8) become infinite. In general we cannot but assume that $\int w_{ik} ds_k$ for large spatial distances r of point i to world line k becomes infinitely small as $1/r$, although this cannot always be realised for individual points; the average value of $\int w_{ik} ds_k$ over a small piece of world line i will then in general surely be $\sim 1/r$. Below we will interpret exceptions to this as quantum-like energy transitions.

It could very well be that the single integral in (7) needs to be generalised and replaced by double integrals, twice over the same world line. It then becomes

$$W = \frac{1}{2} \sum_i \sum_k \iint w_{ik} ds_i ds_k. \quad (10)$$

When the indices are the same, a distinction still has to be made because they both refer to different places on the same world line. Then one can approximately write this as

$$\frac{1}{2} \iint w_{ii'} ds_i ds'_i.$$

However, for straight world lines and longer stretches of straight world lines this has to reduce to $m_i \int ds_i$. In case w_{ii} has particular continuity (and convergence) properties one can write $m_i \int ds_i$ for it anyhow, but with in general a varying m_i dependent upon the acceleration and higher accelerations of particular order. In this work we will not use this generalisation because we will only consider more precisely motions where it [this generalisation] has no influence.

We will now consider one of those motions of n electrons, where they approach from spatially infinite distances, subsequently mutually influence each others trajectories, only to be lost in different directions of infinity afterwards. During the interaction they are not to be influenced significantly by the other electrons of the world. We would like to call this a hyperbolic interaction, and we will assume in accordance with the above that before and after the same [interaction] the integrals with w_{ik} can be neglected so that then $\pi_i = m_i v_i$.

When we multiply the equations (8) with ds_i , integrate over the interaction with infinite boundaries and sum over the n electrons, then we obtain for the change of the total momentum and total energy² $\sum_n m_i v_i^2$ a sum of double integrals over $ds_i ds_k$, that cancel each other pairwise, because

$$\frac{\partial w_{ik}}{\partial Q_i} = -\frac{\partial w_{ik}}{\partial Q_k}. \quad (11)$$

That is why the energy and momentum are ruled by a hyperbolic interaction that is the same as before. In this generalised sense the conservation law for these quantities also holds in our theory. But it does not have the strict form of classical dynamics where it holds for each infinitesimal time element.

It is an essential feature of our theory that the force on an electron is determined not only through the retarded potentials, through the past, but also through the future of the others. However, the two directions of time need not be equal because of this. Because of invariant-theoretic reasons, of which one can be easily convinced, w_{ik} can only depend on the four scalars σ_{ik}^2 , $(v_i \cdot v_k)$, $(v_i \cdot \sigma_{ik})$ and $(v_k \cdot \sigma_{ki})$. Now, in case the two electrons of a pair are of a different sort, then w_{ik} in $(v_i \cdot \sigma_{ik})$ and $(v_k \cdot \sigma_{ki})$ need not be symmetric.

²In the original the square is not well visible, in fact it is only a point, i.e. $\sum_n m_i v_i$; so it could be that $\sum_n m_i v_i$ is meant, although I believe this to be unlikely.

Because the imaginary time-like component of v_i and v_k are always positive imaginary, but $\sigma_{ik} = ic(t_i - t_k)$ and, $\sigma_{ki} = -ic(t_i - t_k)$, and because, in case $\sigma_{ik}^2 < 0$, the sign in front of $(v_i \cdot \sigma_{ik})$ respectively $(v_k \cdot \sigma_{ki})$ is not influenced by the spatial components and therefore corresponds to those with $-(t_i - t_k)$ and respectively $+(t_i - t_k)$, an asymmetry will signify a fundamental distinction between the two directions of time in such a way that the negative electron is influenced mainly or even exclusively by the past of the positive [electron], while the latter is influenced by the future of the first. However, the transformation $t' = -t$ does not belong to the Lorentz group and the laws of nature need be invariant under it. In view of the mass difference of both kinds of electrons, it might be that this inequivalence of both directions of time is very meaningful for the explanation of irreversible phenomena. It is well-known that up until now one first assumes the classical time symmetric dynamical equations, after which a statistical hypothesis, the so-called hypothesis of molecular disorder, is introduced, which implicitly contains a difference between the positive and negative direction of time. Whether this hypothesis is compatible with the classical equations has not been proven; certainly it does not follow from them.

We now investigate the conservation of angular momentum in a hyperbolic interaction. This is easy to proof in the same manner as was done above for momentum and energy when one defines as angular momentum the sum of the vector products $[\varrho_i \cdot \pi_i]$ and realises that members with $[v_i \cdot v_i]$ disappear. One then obtains for the change of angular momentum double integrals with as kernels

$$\left[\sigma_{ik} \cdot \frac{\partial w_{ik}}{\partial \sigma_{ik}} \right] + \left[v_i \cdot \frac{\partial w_{ik}}{\partial v_i} \right] + \left[v_k \cdot \frac{\partial w_{ik}}{\partial v_k} \right]. \quad (12)$$

The sum of these three vector products cancels identically, of which one can convince oneself easily by direct calculation when one considers w_{ik} as a function of the above mentioned four scalars. But usually one considers as angular momentum the quantity $\sum[\varrho_i \cdot m_i v_i]$ and at large distance it is true that, up to order $\sim 1/r$, $m_i v_i = \pi_i$, yet only the ϱ_i become large with r , because of which the equality of $\sum[\varrho_i \cdot \pi_i]$ with $\sum[\varrho_i \cdot m_i v_i]$ is not substantiated offhand. I have found that a sufficient condition for this is the symmetry of w_{ik} with respect to $(v_i \cdot \sigma_{ik})$ and $(v_k \cdot \sigma_{ki})$; however, as far as I can see, this condition is not a necessary one.

Up until now I have not succeeded in discovering those functions w_{ik} that will give the true laws of nature, namely those that give the quanta and I can neither produce the exact proof of their existence; nevertheless, the general considerations that I have been able to present here and because of which the seemingly incomprehensible contradictions of the quantum theory obtain their fundamental solution speaks, in my opinion, very much in favor of the correctness of the assumed principles. Furthermore, their form can be generalised

considerably further, where one only has to presuppose that the laws of nature have to be expressed as functional relations of general form between the world lines of the electrons; therefore the following considerations need not be altered. However, this generalisation will presumably not be necessary.

In classical dynamics one obtains one (and only one) possible motion of a system when one arbitrarily prescribes the coordinates and momenta at a particular point in time; the reason for this is that the time-like differential quotients are because of Hamilton's equations functions of the values of these quantities at the same time. However, according to our equations (8), also the earlier and later values are essentially taken into account, and beforehand it is in general unclear whether it is possible to trace a trajectory through each point of phase space that obeys these equations. In the case of a hyperbolic motion this will probably be the case, because the trajectory goes to infinity at both sides and it seems that one can redirect both branches sufficiently as desired in order to fulfill the equations. However, in the case of quasi-periodic motion it may very well be that the functions w_{ik} have such properties so as to only allow for possible trajectories through particular phase space points, for example that in the Bohrian hydrogen atom only the two motions are possible that are discretely picked out by Sommerfeld's quantum numbers. When this is indeed the case, then, for example, the meeting of two negative electrons and one positive, which produces a hydrogen atom and a free electron, will only be able to produce an atom with integer quantum numbers.

Suppose now that, for example, two atoms of the same kind but with different quantum numbers are located opposite each other at large distance, then they will exert in general only a small and irrelevant effect upon each other; however, at very particular velocities and relative positions, which may be brought about by a brief influence of a third atom (of arbitrary kind), the forces may become very large, somewhat similar to the fact that the time average of $\cos mt \cos nt$ is identically zero, except when $m = n$. It could be that this would then lead to a short lasting energy transition where the atoms would exchange their quantum numbers. The loss of energy of the one and the gain of the other will start at points in time where they are at a corresponding distance from each other, i.e., we are dealing with that which is usually considered to be the emission of radiation and its corresponding absorption. Although according to the classical point of view radiation is simply emitted at random, only to be possibly absorbed somewhere else, according to our theory, however, emission and absorption are processes that require each other, and with each emission it is already predetermined when, where and how the absorption is to be taken place.

The sun would not radiate if it were alone in the cosmos without any other bodies that could absorb its radiation. In our theory there is no fundamental distinction between heat

radiation and heat conductance, and just as impossible it is for a body to lose heat through conduction when the molecules of its surface are not in interaction with those of the other body, just as impossible it is for this through radiation, only the distances are much greater in the last case.

In case the fundamental equations are kept completely general the absorption can occur before the emission, although it seems very unlikely than one can observe this in reality. The absorption can of course also occur in an atom of a different kind, for example through ionization of it, or through combinations of scenarios.

It is also possible that on the light path in between the emitting and absorbing system very numerous and complex systems, for example complete bodies, are located, that take part in the process and will reflect, refract or diffract the radiation. The light, which is emitted as a quantum, will also be absorbed as such, although it could have undergone a very large expansion, as is indeed indicated by the interference phenomena. According to the current theory, which denies the direct relation between absorption and emission, this is incomprehensible and appears quite simply as a logical contradiction.

Yet another result is achieved by the new conception. According to the current view is the emission of a light quantum, by for example a hydrogen atom, given by chance. But one does not at all understand how this comes about. But according to the new theory the emission will take place when it is certain that in another, later world point the absorption will be able to take place in a particular atom or complex of atoms. Accordingly, one can be captivated by the point of view that the amount of matter in the universe is responsible for the omnipresence of the radiation. However, this is not necessarily the case because two competing centres of absorption will not support each other, but will probably hinder each other. When the amount of matter is large enough and somewhat distributed in all directions, then its size will probably not matter. Without any further mathematical investigations a further enquiry into this question seems to be pointless.

When, for example, I have observed through my window some star, which is, say, 100 light years away, then not only I knew that the light which arrived at my eye was emitted by it 100 years ago, but also the star, or at least some of its atoms knew, as it were, already for 100 years, that I, whom at the time did not even exist yet, would observe them yesterday evening at such and such an hour and they also knew something about the dimensions of my window, the place where it is located, the index of refraction of the lenses of my eyes, etc. This sounds quite paradoxical, indeed, because it is contrary to our current habits of thought. The modern development of science has precisely led to the theory of near-by action and the one-sided directed view of causality, which is partly ruled by chance. We know however that

this conception was not originally present in the human mind, and that it was not shared in early times, where it is well-known that very different connections between the phenomena of the world were conjectured, although often in a rather naive way.

In my opinion logically nothing can be objected against our theory. It is in fact nothing but that extension of classical mechanics of discrete mass points as demanded by the relativity principle of the Lorentz transformations.

As far as we can judge without further analysis, probably also experimentally nothing can be said against it. For example, when radiation of the sun is emitted and it is again absorbed on the earth after about eight minutes, then according to classical theory it is present at each instant in time during this intervening period at some particular point in space as field energy. However, our theory has no field, and for it the radiation is lost temporary only to reappear at a later time on the earth. But this does not constitute an observable difference.

Of course, we could place a third body in between the sun and the earth's surface that would absorb part of the radiation, and thereby would, as it were, detect its presence in the space in between, but then this part would not reach the earth's surface so that one cannot say that a ray of radiation has revealed itself on its way. A partly refracting body on the path in between could indeed possibly have some influence, however, we cannot observe this non-absorbing influence there and then. The classical conception appears to be more natural and stems more from our desire for continuousness [continuity/the continuum]; but in my opinion the quantum phenomena force us to principally give it up, although practically it will be needed for simplification of the calculations in many cases (refraction, dispersion, etc.): indeed, in our theory wave optics will not be very clear.

On the last four pages we have pondered quite extensively beyond that which was mathematically exactly proven. Our dynamics contains, just like the classical one, an undetermined force. Because the determination of the form of this force as it actually occurs in nature will probably not be easy and will be put off for at least some time, I felt it to be afforded to complete for the time being the theory via the given general considerations, risking the danger that specific elements of it will appear to be in need of alteration upon closer mathematical investigation. Only in this way it is at all possible to demonstrate the anticipated capability of our point of departure for grasping the quantum phenomena and especially for solving the contradictions between the present theories.