

Analyzing passion at a distance: progress in experimental metaphysics?

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Once again: What does it take to violate Bell's inequality?

Question: What kind of *information*—about the distant measurement setting or the outcome or both—and which amount of it has to be *non-locally available* to simulate the violation of the Clauser–Horne–Shimony–Holt (CHSH) inequality within the framework of hidden-variable models?

To be shown: it is impossible to model a violation without having information in one laboratory about *both* the setting and the outcome at the distant one.

⇒ Progress in **Experimental Metaphysics**

(I) Review of local hidden-variable models

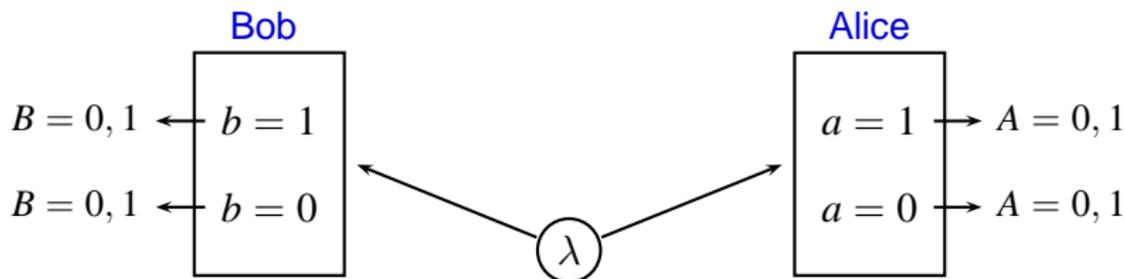
(II) Analyzing the CHSH inequality

- Introducing the Guessed Information (GI)
- Introducing the Transmitted Information (TI)

(III) On what it takes to violate the CHSH inequality – revisited

(IV) Discussion and conclusion

Section 1: Local realism and hidden variables



1. Relevant degrees of freedom are captured in some physical state $\lambda \in \Lambda$ ('beables').
2. The model is concerned with the probabilities $P(A, B|a, b, \lambda)$.
3. Empirically accessible probabilities:

$$P(A, B|a, b) = \int_{\Lambda} P(A, B|a, b, \lambda) \rho(\lambda|a, b) d\lambda.$$

Conditions imposed on the model

1. **Parameter Independence (PI):**

$$P(A|a, b, \lambda) = P(A|a, \lambda) \quad \text{and} \quad P(B|a, b, \lambda) = P(B|b, \lambda).$$

2. **Outcome Independence (OI):**

$$P(A|a, b, B, \lambda) = P(A|a, b, \lambda) \quad \text{and} \quad P(B|a, b, A, \lambda) = P(B|a, b, \lambda).$$

3. **'Freedom of choice':** $\rho(\lambda|a, b) = \rho(\lambda)$.

(also: 'free variables' or 'independence of the source')

Jointly they imply that the CHSH inequality must be obeyed:

$$|\langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle| \leq 2$$

Yet, as is well-known, QM violates this.

Experimental metaphysics

Assuming 'freedom of choice' it must be that either OI or PI is not obeyed in violations of the CHSH inequality

Experimental Metaphysics: extensively argued that it is OI that is to take the blame.

It is then said: Bob, knowing his outcome, can predict Alice's outcome better than was possible just based on the state λ and the settings. But he cannot warn Alice because the outcome is not under his control.

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► This is argued not to be an instance of action at a distance but only of some innocent '**passion at a distance**': *one passively comes to know more about the faraway situation, but one can't actively change it.*

[perhaps more appropriate: "passive at a distance"?)

But this is not passion at a distance at all

Critique:

Both PI and OI do **not** address the possibility of 'coming to know' the *non-local* outcomes or settings.

▶ Violations of PI and OI show a conditional statistical dependence of a *local* probability on a *non-local* outcome or setting.

But, the conditions are not about an increase in non-local predictability because of the availability of non-local information.

Therefore, they do not deal with 'passion at a distance' at all. Such an analysis will be given here.

Section II: Analyzing passion at a distance

[See arXiv:0903.5042¹]

The question to be answered: what kind of information—about the distant measurement setting or the outcome or both—and which amount of it has to be non-locally available to simulate the violation of the CHSH inequality.

¹To appear. Joint work with M. Pawłowski, J. Kofler, Č. Brukner and T. Paterek.

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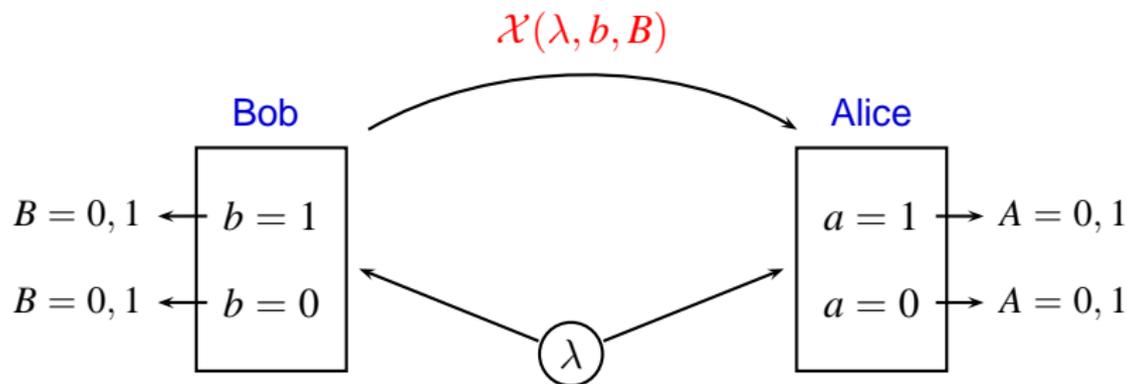
► Here it is assumed that the information becomes available through one-way classical communication.

Although, the results do not depend on there being an actual communication process. *[I will come back to this later.]*

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one-way communication paradigm

Consider the standard Bell-setup, but augmented with one-way classical communication:



$$P(b = 0) = P(b = 1) = \frac{1}{2}$$

$$P(a = 0) = P(a = 1) = \frac{1}{2}$$

one-way communication paradigm

1. Bob generates the *message* \mathcal{X} which depends on λ , b and B .
2. It is assumed that the exact mechanism how B and \mathcal{X} are generated by Bob is known to Alice.
3. Alice uses her *optimal strategy*, based on the knowledge of her setting a , the shared hidden variables λ , and the message \mathcal{X} , to produce her outcome A in order to *maximally* violate the CHSH inequality.

Alternative perspective: 'how nature has to be'
(no longer any reference to Alice's capabilities).

Rewriting the CHSH inequality

The CHSH inequality

$$\langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle \leq 2$$

can be rewritten in terms of joint probabilities $P(A, B|a, b)$ as:

$$\sum_{a,b=0}^1 P(A \oplus B = ab|a, b) \leq 3 \quad (\oplus \text{ mod } 2)$$

e.g., $a, b = 0$ gives: $P(A = 0, B = 0|0, 0) + P(A = 1, B = 1|0, 0)$

Let us now define:

$$P(A = B|a = 0) := \sum_{b'=0}^1 P(b') P(A = B|a = 0, b')$$

$$P(A = B \oplus b|a = 1) := \sum_{b'=0}^1 P(b') P(A = B \oplus b|a = 1, b')$$

and using $P(b = 0) = P(b = 1) = \frac{1}{2}$ allows us to rewrite the CHSH inequality as:

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

This is the **CHSH inequality from Alice's perspective**.

On the 'CHSH inequality from Alice's perspective'

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

These probabilities can be interpreted as a measure of information Alice has about Bob's settings and outcomes.

- To do so, the GuesSED Information Π is introduced:

$$\Pi(\mathcal{X} \rightarrow \mathcal{Y}) := \sum_i P(\mathcal{X} = i) \max_j [P(\mathcal{Y} = j|\mathcal{X} = i)]$$

where \mathcal{X} takes values $i = 1, \dots, X$ and \mathcal{Y} values $j = 1, \dots, Y$.

On the Guessed Information

1. The value of $\Pi(\mathcal{X} \rightarrow \mathcal{Y})$ gives the average probability to correctly guess \mathcal{Y} knowing the value of \mathcal{X} .
2. Its maximal value is 1 and corresponds to the situation in which \mathcal{Y} is fully specified by \mathcal{X} .
3. The minimal value of $\Pi(\mathcal{X} \rightarrow \mathcal{Y})$ equals $\frac{1}{Y}$ and corresponds to the situation in which \mathcal{X} reveals no information about \mathcal{Y} .
4. GI reaches its minimum when the mutual information is $I(\mathcal{X} : \mathcal{Y}) = 0$, and it is maximal when $I(\mathcal{X} : \mathcal{Y}) = \log Y$.

Example:

(i) 'freedom of choice': it must be that $\Pi(\lambda \rightarrow b) = \frac{1}{2}$,

(ii) by contrast, note that $\Pi(\lambda \rightarrow B) > \frac{1}{2}$ is possible.

► The source of the asymmetry between settings and outcomes.

Violating the CHSH inequality

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

Alice must maximize probabilities that not only involve the local information A and a , but also some function $f(B, b)$ containing non-local information.

\implies These probabilities are upperbounded by $\Pi(\lambda, \mathcal{X} \rightarrow f(B, b))$.

This implies the following *necessary condition* for a violation of the CHSH inequality:

$$\frac{1}{2}\Pi(\lambda, \mathcal{X} \rightarrow B) + \frac{1}{2}\Pi(\lambda, \mathcal{X} \rightarrow B \oplus b) > \frac{3}{4}$$

Finally, we are in the position to assess ‘passion at a distance’.

Assessing 'passion at a distance'

The appropriate conditions for assessing 'passion at a distance':

$$\text{Distant Setting Ignorance (DSI): } \Pi(\lambda, \mathcal{X} \rightarrow b) = \frac{1}{2}$$

$$\text{Distant Outcome Ignorance (DOI): } \Pi(\lambda, \mathcal{X} \rightarrow B) = \frac{1}{2}$$

These deal with what can be non-locally predicted. In contrast to OI and PI, they are not about any non-local dependence.

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► **Result:**

A *necessary* condition for violation of CHSH is that *both* information about the setting and about the outcome at one lab *must be available* at the distant lab.

That is, **both of the above conditions must be violated.**

What information must be available, over and above λ ?

One may further ask if

1. the available information comes from the source via the shared hidden variable λ (which acts as a common cause),
 2. or should it be transmitted through the message \mathcal{X} ?
- This calls for a further analysis of what information has to be transmitted via the message \mathcal{X} , over and above the information in the hidden variable λ .

Consider the **Transmitted Information** (TI): the difference of the averaged probability of correctly guessing the value of the variable \mathcal{Y} when knowing \mathcal{X} and λ , and the one when knowing only λ :

$$\Delta_{\lambda}(\mathcal{X} \rightarrow \mathcal{Y}) := \Pi(\lambda, \mathcal{X} \rightarrow \mathcal{Y}) - \Pi(\lambda \rightarrow \mathcal{Y}), \quad \in [0, 1 - \frac{1}{Y}].$$

Its lowest value indicates: transmission of \mathcal{X} does not increase Alice's ability to guess the correct value of \mathcal{Y} .

\implies \mathcal{X} carries no new information about \mathcal{Y} (that is not already available to Alice through λ).

Information Transmission

Distant Setting Transmission (DST): $\Delta_\lambda(\mathcal{X} \rightarrow b) > 0$

Distant Outcome Transmission (DOT): $\Delta_\lambda(\mathcal{X} \rightarrow B) > 0$

- ▶ The 'passion at a distance' not already accounted for by λ .

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In violations of the CHSH inequality:

1. It is possible that the information about the **outcome** can be obtained solely from the shared hidden variables:

It can be that $\Delta_\lambda(\mathcal{X} \rightarrow B) = 0$.

2. However, given 'freedom', the information about the **setting** must be communicated, implicit or explicit, non-locally:

It must be that $\Delta_\lambda(\mathcal{X} \rightarrow b) > 0$.

'Freedom of choice' and the asymmetry

'Freedom of choice' forces this asymmetry between the outcome and setting information:

- It must be that $\Pi(\lambda \rightarrow b) = \frac{1}{2}$.
- Yet no reason to demand $\Pi(\lambda \rightarrow B) = \frac{1}{2}$.

Result:

always $\Delta_\lambda(\mathcal{X} \rightarrow b) \geq 0$

and either $\Delta_\lambda(\mathcal{X} \rightarrow B) = 0$ or $\Pi(\lambda \rightarrow B) = \frac{1}{2}$, but not both.

Section III: Comparing passion and action at a distance

Condition holds	violation of CHSH possible?
$\Pi(\lambda, \mathcal{X} \rightarrow b) = \frac{1}{2}$	No
$\Pi(\lambda, \mathcal{X} \rightarrow B) = \frac{1}{2}$	No
$\Pi(\lambda \rightarrow b) = \frac{1}{2}$	Yes ('freedom')
$\Pi(\lambda \rightarrow B) = \frac{1}{2}$	Yes*
$\Delta_\lambda(\mathcal{X} \rightarrow b) = 0$	No
$\Delta_\lambda(\mathcal{X} \rightarrow B) = 0$	Yes*
OI	Yes**
PI	Yes**

* and **: either one of these conditions can hold, but not both.

Examples

– **Toner and Bacon (2003)**: They simulate the quantum singlet state by communicating 1 classical bit.

New result: If only maximal violation of CHSH is to be simulated, then \mathcal{X} need only contain **0.736 bits**.

– **Leggett-style model of Gröblacher et al. (2007)**: a unit vector is being send.

– **Bohmian mechanics (1952)**: a subtle issue. There is no message sent. But the setting information is non-locally present through the wavefunction which acts as a guiding field.

► In all cases it is setting information which is nonlocally available. (But toy-models can easily be constructed where it is the outcome information which is nonlocally available.)

Section IV: Conclusion and discussion

- 1) All this can be taken out of the one-way communication paradigm. Instead of 'transmission of a message' think of 'extra information being available to Alice'. (e.g. Bohmian mechanics)
- 2) I believe this allows for progress in the field of Experimental Metaphysics.
- 3) As a side effect, it can be noted that these results are also relevant for quantifying the classical resources needed to simulate quantum communication and computation protocols.
- 4) This analysis indicates the asymmetry between outcomes and settings to originate from the 'freedom of choice' assumption.