Analyzing *passion at a distance*: non-local information needed for Bell inequality violation

M.P. Seevinck

Radboud University Nijmegen, The Netherlands

`m.p.seevinck@science.ru.nl`

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Prospects & Introduction

Once again: What does it take to violate Bell's inequality?

Question: What kind of *information*—about the distant measurement setting or the outcome or both—and which amount of it has to be *non-locally available* to account for a violation of the Clauser–Horne–Shimony–Holt (CHSH) inequality within the framework of hidden-variable models?

To be shown: it is impossible to account for a violation without having information in one laboratory about *both* the setting and the outcome at the distant one[1].

$\Longrightarrow$ Progress in **Experimental Metaphysics**

[1] New Journal of Physics, **12** , 083051 (2010). Joint work with M. Pawlowski, J. Kofler, Č. Brukner and T. Paterek.

Methodological morale for this talk:

> *Now it is precisely in cleaning up intuitive ideas for mathematics that one is likely to throw out the baby with the bathwater.*
>
> *J.S. Bell; 'La nouvelle cuisine', 1990.*

Outline

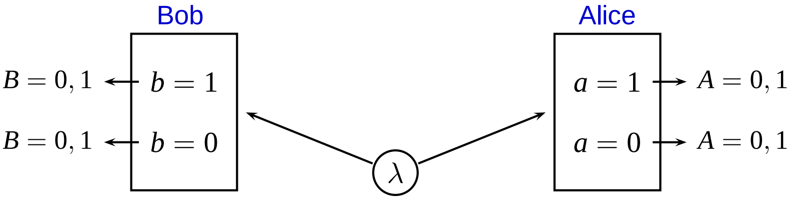
(I) Review of local hidden-variable models

(II) Analyzing the CHSH inequality
- Introducing the Guessed Information (GI)
- Introducing the Transmitted Information (TI)

(III) On what it takes to violate the CHSH inequality – revisited

(IV) Discussion and conclusion

Section I: Local realism and hidden variables



1. Relevant degrees of freedom are captured in some physical state $\lambda \in \Lambda$ (‘beables’).

2. The model is concerned with the probabilities $P(A, B|a, b, \lambda)$.

3. Empirically accessible probabilities:

$$P(A, B|a, b) = \int_\Lambda P(A, B|a, b, \lambda)\rho(\lambda|a, b)d\lambda.$$

Conditions imposed on the model

1. **Parameter Independence** (PI):

$$P(A|a,b,\lambda) = P(A|a,\lambda) \quad \text{and} \quad P(B|a,b,\lambda) = P(B|b,\lambda).$$

2. **Outcome Independence** (OI):

$$P(A|a,b,B,\lambda) = P(A|a,b,\lambda) \quad \text{and} \quad P(B|a,b,A,\lambda) = P(B|a,b,\lambda).$$

3. **‘Free variables’**: $\rho(\lambda|a,b) = \rho(\lambda)$.

   (also called: ’free will’ or ‘measurement independence’)

Jointly they imply that the CHSH inequality must be obeyed:

$$|\langle a_0 b_0\rangle + \langle a_0 b_1\rangle + \langle a_1 b_0\rangle - \langle a_1 b_1\rangle| \leq 2$$

Yet, as is well-known, quantum mechanics violates this.

Experimental metaphysics

Assuming ‘freedom variables’ it must be that either OI or PI is not obeyed in violations of the CHSH inequality

Experimental Metaphysics: extensively argued that it is OI that is to take the blaim, and not PI.

It is then said: Bob, knowing his outcome, can predict Alice’s outcome better than was possible just based on the state λ and the settings. But he cannot warn Alice because the outcome is not under his control.

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- This is argued not to be an instance of action at a distance but only of some innocent ‘passion at a distance’:

  one passively comes to know more about the faraway situation, but one can not actively change it.

[perhaps more appropriate: “passive at a distance”?]

But this is not passion at a distance at all

Critique:

Both PI and OI do **not** address the possibility of 'coming to know' the *non-local* outcomes or settings.

► Violations of PI and OI show a conditional statistical dependence of a *local* probability on a *non-local* outcome or setting.

But, the conditions are not about an increase in non-local predictability because of the availability of non-local information.

Therefore, they do not deal with 'passion at a distance' at all.
Such an analysis will be given here.

Section II: Analyzing passion at a distance

The question to be answered: what kind of information—about the distant measurement setting or the outcome or both—and which amount of it has to be non-locally available to account for a violation of the CHSH inequality[2].

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Section II: Analyzing passion at a distance

The question to be answered: what kind of information—about the distant measurement setting or the outcome or both—and which amount of it has to be non-locally available to account for a violation of the CHSH inequality[2].

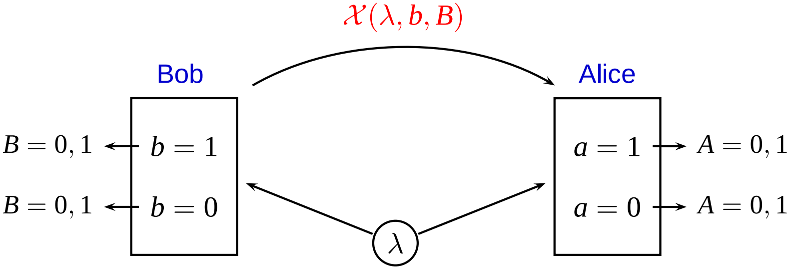
- Here it is assumed that the information becomes available through one-way classical communication.

Although, the results do not depend on there being an actual communication process! [*I will come back to this later*]

[2] New Journal of Physics, **12** , 083051 (2010). Joint work with M. Pawlowski, J. Kofler, Č. Brukner and T. Paterek.

one-way communication paradigm

Consider the standard Bell-setup, but augmented with one-way classical communication:



$P(b = 0) = P(b = 1) = \frac{1}{2}$

$P(a = 0) = P(a = 1) = \frac{1}{2}$

one-way communication paradigm

1. Bob generates the *message* $\mathcal{X}$ which depends on λ, b and B.

2. We allow for Alice to know the exact mechanism how the outcome B and message $\mathcal{X}$ are generated by Bob.

3. Alice uses her *optimal strategy*, based on the knowledge of her setting a, the shared hidden variables λ, and the message $\mathcal{X}$, to produce her outcome A in order to *maximally* violate the CHSH inequality.

Alternative perspective:
– ’how nature has to be’ (no longer any reference to Alice’s capabilities).
– Instead of ’transmission of a message’ think of ’extra information being somehow available to Alice’.

Rewriting the CHSH inequality

The CHSH inequality

$$\langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle \leq 2$$

can be rewritten in terms of joint probabilities $P(A, B|a, b)$ as:

$$\sum_{a,b=0}^{1} P(A \oplus B = ab|a, b) \leq 3 \qquad (\oplus \text{ mod } 2)$$

e.g., $a, b = 0$ gives: $P(A = 0, B = 0|0, 0) + P(A = 1, B = 1|0, 0)$

Let us now define:

$$P(A = B|a = 0) := \sum_{b'=0}^{1} P(b')\, P(A = B|a = 0, b')$$

$$P(A = B \oplus b|a = 1) := \sum_{b'=0}^{1} P(b')\, P(A = B \oplus b|a = 1, b')$$

and using $P(b = 0) = P(b = 1) = \frac{1}{2}$ allows us to rewrite the CHSH inequality as:

$$\frac{1}{2} P(A = B|a = 0) + \frac{1}{2} P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

This is the 'CHSH inequality from Alice's perspective'.

On the 'CHSH inequality from Alice's perspective'

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

These probabilities can be interpreted as a measure of the information Alice has about Bob's settings and outcomes.

- To do so, the Guessed Information Π is introduced:

$$\Pi(\mathcal{X} \to \mathcal{Y}) := \sum_i P(\mathcal{X} = i) \max_j \left[P(\mathcal{Y} = j|\mathcal{X} = i) \right]$$

where $\mathcal{X}$ takes values $i = 1, ..., X$ and $\mathcal{Y}$ values $j = 1, ..., Y$.

On the Guessed Information

1. The value of $\Pi(\mathcal{X} \to \mathcal{Y})$ gives the average probability to correctly guess $\mathcal{Y}$ knowing the value of $\mathcal{X}$.
2. Its maximal value is 1 and corresponds to the situation in which $\mathcal{Y}$ is fully specified by $\mathcal{X}$.
3. The minimal value of $\Pi(\mathcal{X} \to \mathcal{Y})$ equals $\frac{1}{Y}$ and corresponds to the situation in which $\mathcal{X}$ reveals no information about $\mathcal{Y}$.
4. GI reaches its minimum when the mutual information is $I(\mathcal{X} : \mathcal{Y}) = 0$, and it is maximal when $I(\mathcal{X} : \mathcal{Y}) = \log Y$.

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Example: $Y = 2$ (two possible settings and outputs)

(i) ‘free variables’: it must be that $\Pi(\lambda \to b) = \frac{1}{2}$,

(ii) by contrast, note that $\Pi(\lambda \to B) > \frac{1}{2}$ is possible.

- The source of the asymmetry between settings and outcomes.

Violating the CHSH inequality

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

Alice must maximize probabilities that not only involve the local information A and a, but also some function $f(B, b)$ containing non-local information.

$\Longrightarrow$ These probabilities are upperbounded by $\Pi(\lambda, \mathcal{X} \to f(B, b))$.

This implies the following *necessary condition* for a violation of the CHSH inequality:

$$\frac{1}{2}\Pi(\lambda, \mathcal{X} \to B) + \frac{1}{2}\Pi(\lambda, \mathcal{X} \to B \oplus b) > \frac{3}{4}$$

Finally, we are in the position to assess ‘passion at a distance’.

Assessing ‘passion at a distance’

The appropriate conditions for assessing the possibility of ‘coming to know’ the *non-local* outcomes or settings:

$$\text{Distant Setting Ignorance (DSI):} \quad \Pi(\lambda, \mathcal{X} \to b) = \frac{1}{2}$$

$$\text{Distant Outcome Ignorance (DOI):} \quad \Pi(\lambda, \mathcal{X} \to B) = \frac{1}{2}$$

Violations of these deal with what can be non-locally predicted. In contrast to OI and PI, they are not about any non-local dependence.

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- **Result:** A *necessary* condition for violation of CHSH is that *both* information about the setting and about the outcome at one lab *must be available* at the distant lab.

  That is, both of the above conditions must be violated.

PROOF: Both information about the setting and outcome must be available

Necessary for violation: $\frac{1}{2}\Pi(\lambda,\mathcal{X}\to B)+\frac{1}{2}\Pi(\lambda,\mathcal{X}\to B\oplus b)>\frac{3}{4}$

Result: Both information about the setting and outcome must be available.

1. If no outcome information is available, i.e. $\Pi(\lambda,\mathcal{X}\to B)=\frac{1}{2}$, the left-hand side cannot exceed $\frac{3}{4}$.

   $\implies \quad \Pi(\lambda,\mathcal{X}\to B)>\frac{1}{2}$

2. Analogously it must be that $\Pi(\lambda,\mathcal{X}\to B\oplus b)>\frac{1}{2}$.

   To prove that setting information is also necessary, note that if one knows both B and $B\oplus b$, one also knows b.

   This can be made formal: $\Pi(\lambda,\mathcal{X}\to b)>\frac{1}{2}$.

What information must be available, over and above λ?

One may further ask if

1. the available information comes from the source via the shared hidden variable λ (which acts as a common cause),
2. or should it be transmitted through the message $\mathcal{X}$?

► This calls for a further analysis of what information has to be transmitted via the message $\mathcal{X}$, over and above the information in the hidden variable λ.

Information Transmission

Consider the Transmitted Information (TI): the difference of the averaged probability of correctly guessing the value of the variable $\mathcal{Y}$ when knowing $\mathcal{X}$ and λ, and the one when knowing only λ:

$$\Delta_\lambda(\mathcal{X} \to \mathcal{Y}) := \Pi(\lambda, \mathcal{X} \to \mathcal{Y}) - \Pi(\lambda \to \mathcal{Y}), \qquad \in [0, 1 - \frac{1}{Y}].$$

Its lowest value indicates: transmission of $\mathcal{X}$ does not increase Alice's ability to guess the correct value of $\mathcal{Y}$.

$\Longrightarrow$ $\mathcal{X}$ carries no new information about $\mathcal{Y}$ (that is not already available to Alice through λ).

Information Transmission

Distant Setting Transmission (DST): $\Delta_\lambda(\mathcal{X} \to b) > 0$

Distant Outcome Transmission (DOT): $\Delta_\lambda(\mathcal{X} \to B) > 0$

- The 'passion at a distance' not already accounted for by λ.

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- The 'passion at a distance' not already accounted for by λ.

In violations of the CHSH inequality:

1. It is possible that the information about the **outcome** can be obtained solely from the shared hidden variables:

   It <u>can</u> be that $\Delta_\lambda(\mathcal{X} \to B) = 0$.

2. However, given 'free variables', the information about the **setting** must be communicated, implicit or explicit, non-locally:

   It <u>must</u> be that $\Delta_\lambda(\mathcal{X} \to b) > 0$.

‘Free variables’ and the asymmetry

‘Free variables’, i.e $\Pi(\lambda \to b) = \frac{1}{2}$, forces this asymmetry between the outcome and setting information.

(A) It leads to

$$\Delta_\lambda(\mathcal{X} \to b) = \Pi(\lambda, \mathcal{X} \to b) - \frac{1}{2}.$$

1. We see that $\Delta_\lambda(\mathcal{X} \to b) = 0$ leads to $\Pi(\lambda, \mathcal{X} \to b) = \frac{1}{2}$.
2. But we have seen that this implies no violation of the CHSH inequality.
3. Thus the Distant Setting Transmission must be greater than zero: $\Delta_\lambda(\mathcal{X} \to b) > 0$.

‘Freedom of choise’ and the asymmetry

(B) On the other hand, there is no assumption corresponding to ‘free variables’ regarding the outcomes.

- no reason to demand that $\Pi(\lambda \to B) = \frac{1}{2}$.

Instead, one has

$$\Pi(\lambda, \mathcal{X} \to B) = \Delta_\lambda(\mathcal{X} \to B) + \Pi(\lambda \to B).$$

Since $\Pi(\lambda, \mathcal{X} \to B) > \frac{1}{2}$, it can be that either $\Delta_\lambda(\mathcal{X} \to B) = 0$ or $\Pi(\lambda \to B) = \frac{1}{2}$, but not both.

Results: always $\Delta_\lambda(\mathcal{X} \to b) \geq 0$ and either $\Delta_\lambda(\mathcal{X} \to B) = 0$ or $\Pi(\lambda \to B) = \frac{1}{2}$, but not both.

Section III: Comparing passion and action at a distance

Condition holds	violation of CHSH possible?
$\Pi(\lambda, \mathcal{X} \to b) = \frac{1}{2}$	No
$\Pi(\lambda, \mathcal{X} \to B) = \frac{1}{2}$	No
$\Pi(\lambda \to b) = \frac{1}{2}$	Yes ('free variables')
$\Pi(\lambda \to B) = \frac{1}{2}$	Yes*
$\Delta_\lambda(\mathcal{X} \to b) = 0$	No
$\Delta_\lambda(\mathcal{X} \to B) = 0$	Yes*
OI	Yes**
PI	Yes**

* and **: either one of these conditions can hold, but not both.

Examples

– Toner and Bacon (2003): They simulate the quantum singlet state by communicating 1 classical bit.

New result: If only maximal violation of CHSH is to be simulated, then $\mathcal{X}$ need only contain 0.736 bits.

– Leggett-style model of Gröblacher et al. (2007): a unit vector is being send.

– Bohmian mechanics (1952): a subtle issue. There is no message sent. But the setting information is non-locally present through the wavefunction which acts as a guiding field.

► In all cases it is setting information which is nonlocally available. (But toy-models can easily be constructed where it is the outcome information which is nonlocally available.)

Section IV: Conclusion and discussion

1) A CHSH violation requires that information about both the outcome and the setting at one laboratory is available at the distant one, despite the fact that there is *no need for both* non-local Setting and Outcome Dependence (¬OI, ¬ PI).

2) All this can be taken out of the one-way communication paradigm. Instead of 'transmission of a message' think of 'extra information being available to Alice'. (e.g. Bohmian mechanics)

3) The necessity that –within hidden variable models and 'free variables'– information about freely chosen distant settings has to be available in a space-like separated way seriously questions the possibility of Lorentz invariant completion of quantum mechanics.

4) This analysis indicates the asymmetry between conditions on outcomes and on settings (*OI vs. PI, etc.*) to originate from the 'free variables' assumption.