Analyzing passion at a distance: progress in experimental metaphysics?

M.P. Seevinck

Institute for Theoretical Physics
&
Institute for History and Foundations of Science

Utrecht University, The Netherlands

m.p.seevinck@uu.nl

October 2009

Prospects & Introduction

Once again: What does it take to violate Bell's inequality?

Question: What kind of *information*—about the distant measurement setting or the outcome or both—and which amount of it has to be *non-locally available* to simulate the violation of the Clauser–Horne–Shimony–Holt (CHSH) inequality within the framework of hidden-variable models?

To be shown: it is impossible to model a violation without having information in one laboratory about *both* the setting and the outcome at the distant one.

$\Longrightarrow$ Progress in **Experimental Metaphysics**

Outline

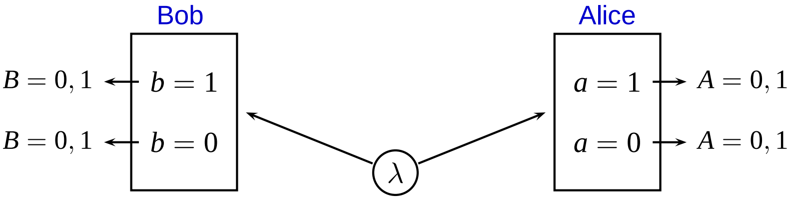
(I) Review of local hidden-variable models
- outcome independence (OI) and parameter independence (PI)
- experimental metaphysics and action vs. passion at a distance

(II) Analyzing passion at a distance
- Introducing the Guessed Information (GI)
- Introducing the Transmitted Information (TI)

(III) On what it takes to violate the CHSH inequality revisited

(IV) Discussion and conclusion

Section I: Local realism and hidden variables



1. Relevant degrees of freedom are captured in some physical state $\lambda \in \Lambda$ (‘beables’).

2. The model is concerned with the probabilities $P(A, B|a, b, \lambda)$.

3. Empirically accessible probabilities:

$$P(A, B|a, b) = \int_\Lambda P(A, B|a, b, \lambda)\rho(\lambda|a, b)d\lambda.$$

Conditions imposed on the model

1. **Parameter Independence** (PI):

   $$P(A|a,b,\lambda) = P(A|a,\lambda) \quad \text{and} \quad P(B|a,b,\lambda) = P(B|b,\lambda).$$

2. **Outcome Independence** (OI):

   $$P(A|a,b,B,\lambda) = P(A|a,b,\lambda) \quad \text{and} \quad P(B|a,b,A,\lambda) = P(B|a,b,\lambda).$$

3. **'Freedom of choice'**: $\rho(\lambda|a,b) = \rho(\lambda)$.

   (also: 'free variables' or 'independence of the source')

Jointly they imply that the CHSH inequality must be obeyed:

$$|\langle a_0 b_0\rangle + \langle a_0 b_1\rangle + \langle a_1 b_0\rangle - \langle a_1 b_1\rangle| \leq 2$$

Yet, as is well-known, QM violates this.

Experimental metaphysics

Assuming 'freedom of choice' it must be that either OI or PI is not obeyed in violations of the CHSH inequality

Experimental Metaphysics: it is OI that is to take the blaim:

$\neg$ OI: $P(A|a, b, B, \lambda) \neq P(A|a, b, \lambda)$.

It is then said: Bob, knowing his outcome, can predict Alice's outcome better than was possible just based on the state λ and the settings. But he cannot warn Alice because the outcome is not under his control.

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- It is argued that this is not an instance of action at a distance but of some innocent ‘passion at a distance’: one passively comes to know more about the faraway situation, but one cannot actively change it.

But this is not passion at a distance at all

$\neg$ PI: $$P(A|a, b, \lambda) \neq P(A|a, \lambda)$$

$\neg$ OI: $$P(A|a, b, B, \lambda) \neq P(A|a, b, \lambda)$$

Both PI and OI do **not** address the possibility of ‘coming to know’ the *non-local* outcomes or settings.

- Violations of PI and OI show a dependence of a *local* probability on a *non-local* outcome or setting.

More technically: the conditions are not about an increase in non-local predictability because of the availability of non-local information.

Therefore, they do not deal with passion at a distance at all. Such an analysis will be given here.

Section II: Analyzing passion at a distance

[Joint work with Pawlowski et al. (arXiv:0903.5042)]

The question to be answered: what kind of information—about the distant measurement setting or the outcome or both—and which amount of it has to be non-locally available to simulate the violation of the CHSH inequality.

Section II: Analyzing passion at a distance

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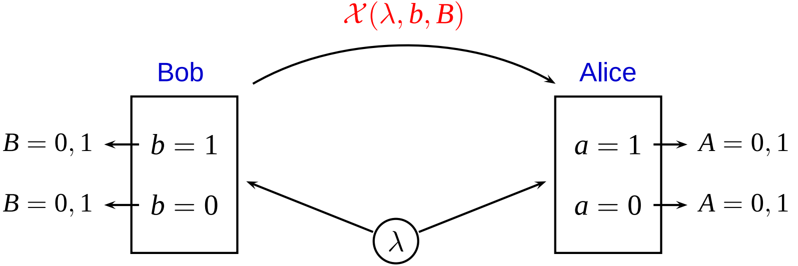
The question to be answered: what kind of information—about the distant measurement setting or the outcome or both—and which amount of it has to be non-locally available to simulate the violation of the CHSH inequality.

- Here it is assumed that the information becomes available through one-way classical communication.

Although, the results do not depend on there being an actual communication process.

one-way communication paradigm

Consider the standard Bell-setup, but augmented with one-way classical communication:



$P(b = 0) = P(b = 1) = \frac{1}{2}$

$P(a = 0) = P(a = 1) = \frac{1}{2}$

one-way communication paradigm

1. Bob generates the *message* $\mathcal{X}$ which depends on λ, b and B.
2. It is assumed that the exact mechanism how B and $\mathcal{X}$ are generated by Bob is known to Alice.
3. Alice uses her *optimal strategy*, based on the knowledge of her setting a, the shared hidden variables λ, and the message $\mathcal{X}$, to produce her outcome A in order to *maximally* violate the CHSH inequality.

Alternative perspective: 'how nature has to be'

(no longer any reference to Alice's capabilities).

Rewriting the CHSH inequality

The CHSH inequality

$$\langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle \leq 2$$

can be rewritten in terms of joint probabilities $P(A, B|a, b)$ as:

$$\sum_{a,b=0}^{1} P(A \oplus B = ab|a, b) \leq 3 \qquad (\oplus \text{ mod } 2)$$

e.g., $a, b = 0$ gives: $P(A = 0, B = 0|0, 0) + P(A = 1, B = 1|0, 0)$

Let us now define:

$$P(A = B|a = 0) := \sum_{b'=0}^{1} P(b')\, P(A = B|a = 0, b')$$

$$P(A = B \oplus b|a = 1) := \sum_{b'=0}^{1} P(b')\, P(A = B \oplus b|a = 1, b')$$

and using $P(b = 0) = P(b = 1) = \frac{1}{2}$ allows us to rewrite the CHSH inequality as:

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

This is the CHSH inequality from Alice's perspective.

On the ‘CHSH inequality from Alice’s perspective’

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

These probabilities can be interpreted as a measure of information Alice has about Bob’s settings and outcomes.

- To do so, the Guessed Information Π is introduced:

$$\Pi(\mathcal{X} \to \mathcal{Y}) := \sum_i P(\mathcal{X} = i) \max_j \left[P(\mathcal{Y} = j|\mathcal{X} = i) \right]$$

where $\mathcal{X}$ takes values $i = 1, ..., X$ and $\mathcal{Y}$ values $j = 1, ..., Y$.

On the Guessed Information

1. The value of $\Pi(\mathcal{X} \rightarrow \mathcal{Y})$ gives the average probability to correctly guess $\mathcal{Y}$ knowing the value of $\mathcal{X}$.
2. Its maximal value is 1 and corresponds to the situation in which $\mathcal{Y}$ is fully specified by $\mathcal{X}$.
3. The minimal value of $\Pi(\mathcal{X} \rightarrow \mathcal{Y})$ equals $\frac{1}{Y}$ and corresponds to the situation in which $\mathcal{X}$ reveals no information about $\mathcal{Y}$.
4. GI reaches its minimum when the mutual information is $I(\mathcal{X} : \mathcal{Y}) = 0$, and it is maximal when $I(\mathcal{X} : \mathcal{Y}) = \log Y$.

Example:

(i) ‘freedom of choice’: it must be that $\Pi(\lambda \rightarrow b) = \frac{1}{2}$,

(ii) by contrast, note that $\Pi(\lambda \rightarrow B) > \frac{1}{2}$ is possible.

- The source of the asymmetry between settings and outcomes.

Violating the CHSH inequality

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}$$

Alice must maximize probabilities that not only involve the local information A and a, but also some function $f(B, b)$ containing non-local information.

$\Longrightarrow$ These probabilities are upperbounded by $\Pi(\lambda, \mathcal{X} \to f(B, b))$.

This implies the following *necessary condition* for a violation of the CHSH inequality:

$$\frac{1}{2}\Pi(\lambda, \mathcal{X} \to B) + \frac{1}{2}\Pi(\lambda, \mathcal{X} \to B \oplus b) > \frac{3}{4}$$

Finally, we are in the position to assess ‘passion at a distance’.

Assessing ‘passion at a distance’

$$\text{Distant Setting Ignorance (DSI):} \quad \Pi(\lambda, \mathcal{X} \to b) = \frac{1}{2}$$

$$\text{Distant Outcome Ignorance (DOI):} \quad \Pi(\lambda, \mathcal{X} \to B) = \frac{1}{2}$$

These deal with what can be non-locally predicted. In contrast to OI and PI, they are not about any non-local dependence.

- The appropriate conditions for assessing ‘passion at a distance’.

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- The appropriate conditions for assessing ‘passion at a distance’.

A *necessary* condition for violation of CHSH is that *both* information about the setting and about the outcome at one lab *must be available* at the distant lab.

That is, both of the above conditions must be violated.

Both information about the setting and outcome must be available

Necessary for violation: $\frac{1}{2}\Pi(\lambda, \mathcal{X} \to B) + \frac{1}{2}\Pi(\lambda, \mathcal{X} \to B \oplus b) > \frac{3}{4}$

Both information about the setting and outcome must be available.

1. If no outcome information is available, i.e. $\Pi(\lambda, \mathcal{X} \to B) = \frac{1}{2}$, the left-hand side cannot exceed $\frac{3}{4}$.

   $\implies \quad \Pi(\lambda, \mathcal{X} \to B) > \frac{1}{2}$

2. Analogously it must be that $\Pi(\lambda, \mathcal{X} \to B \oplus b) > \frac{1}{2}$.

   To prove that setting information is also necessary, note that if one knows both B and $B \oplus b$, one also knows b.

   This can be made formal: $\Pi(\lambda, \mathcal{X} \to b) > \frac{1}{2}$.

What information must be available, over and above λ?

One may further ask if

1. the available information comes from the source via the shared hidden variable λ (which acts as a common cause),
2. or should it be transmitted through the message $\mathcal{X}$?

► This calls for a further analysis of what information has to be transmitted via the message $\mathcal{X}$, over and above the information in the hidden variable λ.

Information Transmission

Consider the Transmitted Information (TI): the difference of the averaged probability of correctly guessing the value of the variable $\mathcal{Y}$ when knowing $\mathcal{X}$ and λ, and the one when knowing only λ:

$$\Delta_\lambda(\mathcal{X} \to \mathcal{Y}) := \Pi(\lambda, \mathcal{X} \to \mathcal{Y}) - \Pi(\lambda \to \mathcal{Y}), \qquad \in [0, 1 - \frac{1}{Y}].$$

Its lowest value indicates: transmission of $\mathcal{X}$ does not increase Alice's ability to guess the correct value of $\mathcal{Y}$.

$\Longrightarrow$ $\mathcal{X}$ carries no new information about $\mathcal{Y}$ (that is not already available to Alice through λ).

Information Transmission

Distant Setting Transmission (DST): $\Delta_\lambda(\mathcal{X} \rightarrow b) > 0$

Distant Outcome Transmission (DST): $\Delta_\lambda(\mathcal{X} \rightarrow B) > 0$

- The ’passion at a distance’ not already accounted for by λ.

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- The 'passion at a distance' not already accounted for by λ.

In violations of the CHSH inequality:

1. It is possible that the information about the **outcome** can be obtained solely from the shared hidden variables:

   It <u>can</u> be that $\Delta_\lambda(\mathcal{X} \to B) = 0$.

2. However, given 'freedom', the information about the **setting** must be communicated, implicit or explicit, non-locally:

   It <u>must</u> be that $\Delta_\lambda(\mathcal{X} \to b) > 0$.

‘Freedom of choise’ and the asymmetry

‘Freedom of choice’ forces this asymmetry between the outcome and setting information:

- It must be that $\Pi(\lambda \to b) = \frac{1}{2}$.
- Yet no reason to demand $\Pi(\lambda \to B) = \frac{1}{2}$.

Result:

always $\Delta_\lambda(\mathcal{X} \to b) \geq 0$

and either $\Delta_\lambda(\mathcal{X} \to B) = 0$ or $\Pi(\lambda \to B) = \frac{1}{2}$, but not both.

Section III: Comparing passion and action at a distance

Condition holds	violation of CHSH possible?
$\Pi(\lambda, \mathcal{X} \to b) = \frac{1}{2}$	No
$\Pi(\lambda, \mathcal{X} \to B) = \frac{1}{2}$	No
$\Pi(\lambda \to b) = \frac{1}{2}$	Yes ('freedom')
$\Pi(\lambda \to B) = \frac{1}{2}$	Yes*
$\Delta_\lambda(\mathcal{X} \to b) = 0$	No
$\Delta_\lambda(\mathcal{X} \to B) = 0$	Yes*
OI	Yes**
PI	Yes**

* and **: either one of these conditions can hold, but not both.

Examples

– Toner and Bacon (2003): They simulate the quantum singlet state by communicating 1 classical bit.

New result: If only maximal violation of CHSH is to be simulated, then $\mathcal{X}$ need only contain 0.736 bits.

– Leggett-style model of Gröblacher et al. (2007): a unit vector is being send.

– Bohmian mechanics (1952): a subtle issue. There is no message sent. But the setting information is non-locally present through the wavefunction which acts as a guiding field.

► In all cases it is setting information which is nonlocally available.

Section IV: Conclusion and discussion

1) All this can be taken out of the one-way communication paradigm. Instead of ’transmission of a message’ think of ’ extra information being available to Alice’. (e.g. Bohmian mechanics)

2) I believe this allows for progress in the field of Experimental Metaphysics.

3) As a side effect, it can be noted that these results are also relevant for quantifying the classical resources needed to simulate quantum communication and computation protocols.

4) This analysis tried to trace the asymmetry between outcomes and settings so as to originate from the ‘freedom of choice’ assumption.