# What is beneath the surface?

## On deeper level hidden-variables and comparing surface and subsurface levels

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# Prospects & Introduction

I will reconsider the well-known (local) hidden variable program.

Some elementary investigations and new results are presented that have general repercussions. These are intended to deepen our understanding of what it takes to violate local realism and/or the CHSH inequality and how this relates to signaling vs. no-signaling correlations.

> *Now it is precisely in cleaning up intuitive ideas for mathematics that one is likely to throw out the baby with the bathwater.*
>
> *J.S. Bell; ‘La nouvelle cuisine’, 1990.*

# Outline

PART 1

(I) Review of (local) hidden-variable models

On what it takes to obey the CHSH inequality: a moral story

(II) Incomplete hidden-variable descriptions

Consequences of the existence of a deeper level

PART 2

(III) Further Definitions

(IV) Surface vs. subsurface level

Determinism, yet indeterminism

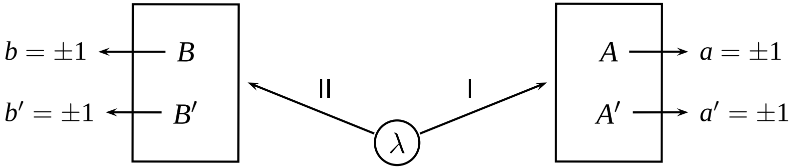
(V) Discerning no-signaling correlations

The CHSH inequality in disguise

(VI) Conclusion and outlook

# Local realism and hidden variables

Setup of the *Gedankenexperiment*:




- Locality: the idea that there exists no spacelike causation.
- Realism: the idea that (i) physical systems exist independently and (ii) posses intrinsic properties describable by states.
- Free variables: the settings used to measure observables can be chosen freely, i.e., this excludes conspiracy theories (e.g., super-determinism) as well as retro-causal interactions.

1. One assumes that the particle pair is described by some physical state $\lambda \in \Lambda$ ('beables').

2. Further requirement that is often used: $\lambda$ provides a complete (or *full* or *total* or *exhaustive*) specification of the state of the system in question.

   $\rightarrow$ In need of clarification . . . to be continued.

3. The hidden variable model gives the probability for obtaining outcomes $a, b$ when measuring $A, B$ on a system in the state $\lambda$:

$$P(a, b|A, B, \lambda).$$

4. Empirically accessible probabilities of outcomes are obtained by averaging over some probability density on $\lambda$:

$$P(a, b|A, B) = \int_\Lambda P(a, b|A, B, \lambda)\rho(\lambda|A, B)d\lambda.$$

# Conditions

– Independence of the Source (IS): $\rho(\lambda|A,B) = \rho(\lambda)$.

– Parameter Independence (PI):

$$P(a|A,B,\lambda) = P(a|A,\lambda) \text{ and } P(b|A,B,\lambda) = P(b|B,\lambda).$$

– Outcome Independence (OI):

$$P(a,b|A,B,\lambda) = P(a|A,B,\lambda)P(b|A,B,\lambda).$$

- Motivations for these conditions:

IS: via the notion of free variables.

PI: via invoking locality.

OI: opinions differ. Some use locality, others rely on realism or invoke some other idea.

# Motivating OI: $P(a, b|A, B, \lambda) = P(a|A, B, \lambda)P(b|A, B, \lambda)$

I take OI to be a completeness or sufficiency condition that encodes (partly) the realism assumption.

- $\lambda$ (together with the settings $A$ and $B$) is complete, i.e., sufficient for the (probability of obtaining) outcomes $a$ and $b$. To be further clarified later.

- OI is not to be motivated by locality. But this is controversial.

– Shimony uses an appeal to locality, albeit different from superluminal signaling.
– Elby, Brown and Foster claim that 'Jarrett completeness [OI] follows from natural assumptions about locality and causality'.
– Others use Reichenbach's principle of common cause or Helmann's Stochastic Einstein Locality (cf. Butterfield).
– Norsen: 'The whole motivation of Jarrett's project . . . is based on a fundamental confusion'.

# Consequences

PI $\wedge$ OI $\implies$ Factorisability (Bell called this ‘local causality’):

$$P(a,b|A,B,\lambda) = P(a|A,\lambda)P(b|B,\lambda).$$

Factorisability $\wedge$ IS $\implies P(a,b|A,B) = \int_\Lambda P(a|A,\lambda)P(b|B,\lambda)\rho(\lambda)d\lambda$

$\implies$ CHSH inequality is obeyed.

## Intermezzo

The CHSH inequality can still be derived after relaxing conditions IS, PI and OI in specific ways. Indeed, violations of these conditions are allowed for (MS, 2008).

● Therefore a larger class of hidden variable theories than is usually considered is ruled out by quantum theory and modulo some loopholes also by experiment.

► No set of necessary and sufficient conditions for deriving the CHSH inequality is known, and consequently we do not precisely know what a violation of the inequality amounts to.

It is important to recognize this if we are to have a proper appreciation of the epistemological situation we are in when we attempt to glean metaphysical implications of the failure of the CHSH inequality.

# Completeness and deeper hidden variables

Consider a theory that posits that apart from the hidden variable $\lambda$ there is also a deeper level hidden variable $\mu$ and that all the probabilities $P(a,b|A,B,\lambda)$ are actually averages over the additional variable:

$$P(a,b|A,B,\lambda) = \int P(a,b|A,B,\lambda,\mu)\,\rho(\mu|\lambda)\,d\mu$$

Mathematically this is always possible and one can go as deep as to get a fully deterministic theory: all $P(a,b|A,B,\lambda,\mu)$ are either 0 or 1.

- Suppose a deeper level exists. If conditions hold at one level, must they also hold at another level? NO!

# Examples

**(A)**

- Orthodox QM: $\lambda = |\,\psi\rangle \implies$ PI holds, OI fails
- Bohmian mechanics: $\lambda = (|\,\psi\rangle, \vec{x}_1, \vec{x}_2) \implies$ OI holds, PI fails.
  (deeper level hidden variables, deterministic.)

**(B)** Leggett's 2003 model:

- Deepest deterministic level: $\lambda = (\gamma, \vec{u}, \vec{v}) \implies$ PI fails, OI holds
- On the level of $(\vec{u}, \vec{v})$; average over $\gamma \implies$ OI fails, PI holds.

(Note: in Leggett's model $\gamma$ does no work at all. All his physical assumptions are at the $(\vec{u}, \vec{v})$ level.)

This shows explicitly that parameter dependence (violation of PI) at the deeper deterministic hidden-variable level does not show up as parameter dependence at the higher hidden-variable level, but as setting dependence, i.e., as a violation of OI.

In other words, violation of OI could be a sign of a violation of (deterministic) PI at a deeper hidden-variable level.

► In the deterministic case the feature above is generic: a violation of OI implies a violation of deterministic PI at the deeper hidden-variable level where the model is deterministic.

# General picture

| Higher level | | Deeper level |
| --- | --- | --- |
| PI holds | $\not\Rightarrow$ <br> $\Leftarrow$ | PI holds |
| OI holds | $\not\Rightarrow$ <br> $\not\Leftarrow$ | OI holds |

PI at deeper level implies PI at higher level.

- Independence at a lower level is conserved by averaging. Averaging cannot create any dependencies.

OI at deeper level does not imply OI at higher level.

- If regarded as a completeness condition this is to be expected since one generally leaves out some relevant hidden variables. Completeness then is given up.

**Cause of the assymetry**: Mathematically:

- Due to the non-convexity of factorisation conditions such as OI. They do no longer hold under convex combinations and decompositions.
- Whereas: independence conditions such as PI remain to hold under convex combinations.

| Higher level | | Deeper level |
|---|---|---|
| PI holds | $\overset{\not\Longrightarrow}{\Longleftarrow}$ | PI holds |
| OI holds | $\overset{\not\Longrightarrow}{\not\Longleftarrow}$ | OI holds |

Analogous to OI (b redundant): $P(a|A, B, b, \lambda) = P(a|A, B, \lambda)$.
The idea is that extra hidden-variables become redundant.
**Completeness***: $\lambda$ is complete if $P(a, b|A, B, \lambda, \xi) = P(a, b|A, B, \lambda)$ for all $a, b, A, B$ and all possible extra hidden variables $\xi$.

- If Completeness* holds, then:

| | | |
|---|---|---|
| OI holds | $\Longrightarrow$ | OI holds |
| PI holds | $\Longrightarrow$ | PI holds |

- Note that Completeness* does not force determinism. This formalises the notion of ‘a *complete* hidden-variable model’ in a probabilistic framework.

# Being explicit about completeness

We thus see that which conditions are obeyed and which are not depends on the level of consideration.

- A conclusive picture therefore depends on which hidden-variable level is considered to be fundamental.

But usually this is not mentioned. This is unfortunate, it hinders interpretation.

Many use such words as *complete, full, exhaustive, sufficient.* But none give a specific definition, and moreover any such a model can be considered to give a complete specification, namely of all variables that happen to feature in the model.

- But this misses the point. The point is whether the hidden variables are Complete*.

Bell stressed the importance of ‘completeness’.

“A theory is said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by a specification of values of local beables in a space-like separated region 2 when what happens in the backward light cone is already **sufficiently** specified, for example by a full specification of local beables in a spacetime region 3. It is important that region 3 completely shields off from 1 the overlap of the backward light cones of 1 and 2. And it is important that events 3 be **specified completely**. Otherwise the traces in region 2 of causes of events in 1 could well supplement whatever else was being used for calculating probabilities about 1. **The hypothesis is that any such information about 2 becomes redundant when 3 is specified completely**.”

(Bell, La Nouvelle cuisine, p.106. Emphasis added.)

“Invoking local causality and **the assumed completeness of $c$ and $\lambda$**, ... we have $P(a,b|A,B,\lambda,c) = P(a|A,\lambda,c)P(b|B,\lambda,c)$”

(Ibid, p.109, emphasis added)

# Conclusion and Discussion (of Part 1)

- The strategy employed:

Investigate the consequences of the possibility and relevance of extra hidden variables at a deeper level.

- One should not just consider the structural form of the probabilistic conditions, but also what is said about the completeness of the hidden variable specification at a certain level.

- The condition of Completeness$^*$ captures this aspect. Whether or not it holds should generally be addressed in any hidden variable program.

- Open question: What happens if we bring in quantum theory? What are the repercussions of the requirement that one should also reproduce QM?

# Part 2: Further definitions

**Surface probabilities**: $P(a,b|A,B)$
Determined via measurement of relative frequencies.

**Subsurface probabilities**: $P(a,b|A,B,\lambda)$
Generally inaccessible, conditioned on hidden variables.

- Definitions of different kinds of bi-partite surface correlations:

a) **Local**: $P(a,b|A,B) = \int_\Lambda d\lambda\, \rho(\lambda)\, P(a|A,\lambda)P(b|B,\lambda)$.
(non-local = not local)

b) **No-signaling**: $P(a|A)^B = P(a|A)^{B'} := P(a|A)$
where $P(a|A)^B = \sum_b P(a,b|A,B)$, etc.

[ c) **Quantum**: $P(a,b|A,B) =$Tr$[\, M_a^A \otimes M_b^B\, \rho\,]$, $\sum_a M_a^A = \mathbb{1}, \forall A$. ]

# Surface vs. Subsurface Levels

**Subsurface:**

- OI $\wedge$ PI $\implies$ Factorisability
- Determinism $\implies$ OI

(i) Deterministic hidden variables and violation of Factorisability implies violation of PI. (e.g. Bohmian mechanics)

(ii) PI and violation of Factorisability implies indeterminism at the hidden-variable level.

**Surface analogs of (i) and (ii):**

(iii) Any non-local correlation that is deterministic must be signaling.

(iv) Any non-local correlation that is no-signaling must be indeterministic, i.e., the outcomes are only probabilistically determined.

**Proof:** Any deterministic no-signaling correlation must be local. [Masanes et al. (2006)]

- Consider a deterministic probability distribution $P_{\text{det}}(a,b|AB)$.

$\Longrightarrow$ The outcomes $a$ and $b$ are deterministic functions of $A$ and $B$: $a = a[A,B]$ and $b = b[A,B]$.

- Suppose it is a no-signaling distribution, then

$$P_{\text{det}}(a,b|AB) = \delta_{(a,b),(a[(A,B],b[A,B])} = \delta_{a,a[A,B]}\delta_{b,b[A,B]}$$
$$= P(a|A,B)P(b|A,B) = P(a|A)P(b|B).$$

This is a local distribution and therefore any deterministic no-signaling correlation must be local.

# Determinism, yet indeterminism

Now again consider Bohmian mechanics: because it obeys no-signaling and gives rise to non-local correlations (since it violates the CHSH inequality) it must determine the outcomes only probabilistically.

In other words, although fundamentally deterministic it must necessarily be predictively indeterministic.

▶ Thus no Bohmian demon can have perfect control over the hidden variables and still be non-local and no-signaling at the surface (as QM requires).

• This is not specific to Bohmian mechanics: any deterministic theory that obeys no-signaling and gives non-local correlations must have the same feature: it must determine the outcomes of measurement indeterministically.

# Discerning no-signaling correlations

We have seen that requiring no-signaling in conjunction with some other constraint has strong consequences.

- But what if we solely require no-signaling? Can we find a non-trivial constraint that follows from no-signaling alone?

The CHSH inequality does not suffice to discern no-signaling correlations because they can maximally violate it up to a value of 4 (e.g., PR-boxes). *But an analogon does*:

$$|\langle AB\rangle + \langle A'B\rangle + \langle A\rangle^B - \langle A'\rangle^B| \leq 2.$$

Here $\langle A\rangle^B := \sum_{a,b} a \int_\Lambda d\lambda \rho(\lambda|A,B)P(a,b|A,B,\lambda)$. This can contain any non-local or signaling dependencies on the setting $A$ and $B$.

# Reproducing perfect singlet-state correlations

$$\forall\,\vec{a},\,\vec{b}:\quad \langle \vec{a}\,\vec{b}\rangle = -1,\quad \text{when}\;\; \vec{a} = \vec{b}$$
$$\forall\,\vec{a},\,\vec{b}:\quad \langle \vec{a}\,\vec{b}\rangle = 1,\qquad \text{when}\;\; \vec{a} = -\vec{b}$$

- The no-signaling inequalities give two non-trivial constraints:

$$\langle \vec{a}\rangle^{I}_{\text{ns}} + \langle \vec{a}\rangle^{II}_{\text{ns}} = 0$$
$$\langle -\vec{a}\rangle^{I}_{\text{ns}} = -\langle \vec{a}\rangle^{I}_{\text{ns}}$$

This states that the marginal expectation values for party $I$ and $II$ must add up to zero for measurements in the same direction, and individually they must be odd functions of the settings.

- Consequently, any model reproducing the singlet state perfect (anti-) correlations and which does not obey either one (or both) of these conditions must be signaling.

► In case both systems are treated the same, i.e., $\langle \vec{a}\rangle^{I}_{\text{ns}} = \langle \vec{a}\rangle^{II}_{\text{ns}}$, the marginal expectation values must vanish: $\langle \vec{a}\rangle^{I}_{\text{ns}} = \langle \vec{a}\rangle^{II}_{\text{ns}} = 0$.

# Conclusion and Discussion

- Two strategies have been employed:

i) An investigation of how inferences that hold on the surface level relate to those those that hold on the subsurface level.

ii) Investigate the consequences of the possibility and relevance of extra hidden variables at a deeper level.

- One should not just consider the structural form of the probabilistic conditions, but also what is said about the completeness of the hidden variable specification at a certain level.

- The condition of Completeness* captures this aspect. Whether or not it holds should generally be addressed in any hidden variable program.

# Conclusion and Discussion

- Open question: What happens if we bring in quantum theory? What are the repercussions of the requirement that one should also reproduce QM?

Only reproducing perfect singlet correlations was considered, and only for no-signaling correlations.

(Valentini's result: any deterministic HV theory that gives QM for some equilibrium distribution (of the HV's) must be signaling for some non-equilibrium distribution).