STRUCTURALISM IN PHYSICS

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Talk based on various papers in: *British Journal for the Philosophy of Science*, *Philosophy of Science*, *Synthese*; and on submissions to: *Mind*, *Journal of Philosophy*;

see websites authors.

Overview

0. Structuralism in Realism Debate in Philosophy of Science

1. Structuralism in Physics

2. Quantum Mechanics

3. The General and Special Theory of Relativity

4. Connexions to Metaphysics and Philosophy of Mathematics

Appendix: Quantum Field Theory

Early Birds

Henri Poincaré, Bertrand Russell, Grover Maxwell

John Worrall re-discovered this view and expounded it as a variety of scientific realism, so as

(A) to avoid falling prey to the pessimistic meta-induction over the history of science
[most powerful argument against realism];

whilst remaining able

(B) to explain the empirical and technological success of science
[to make that success not a miracle —
most powerful argument in favour of realism].

locus classicus

J. Worrall, 'Structural Realism: The Best of Both Worlds?',
Dialectica **43** (1989) 99–124.

Structuralism (Structural Realism)

What is it? What exactly does it commit one to?

Worrall's Slogan:

Science discovers the structure of the world, not its nature.

J. Ladyman: distinghuish **epistemic** and **ontic** structuralism.

Is it the best variety of scientific realism?

Enter the Realism Debate.

One point of consensus: if so, then certainly for **physics**.

So ... if **not** for physics, then farewell **Structuralism**.

Some arguments in favour of Structuralism currently discussed:

- The Argument from Perception.
- The Argument from the History of Science.
- The Argument from Transmission.
- The Argument from Mathematical Representation.
- The Argument from Predictive Power.
- The Argument from Perfect Alignment.
- The Argument from Epistemology.
- The Argument from Symmetry Groups.
- **The Argument from Quantum Mechanics.**
- **The Argument from Space-Time.**
- **The Argument from Quantum Field Theory** (if time permits).

Structuralism about physical theory T

0. **Principle of Characterisation of T.**
   T is characterised through the set of **T**-structures $\mathfrak{S}$.
1. **Principle of Automorphic Properties and Relations.**
   Only **T**-automorphic subsets represent properties and relations.
2. **Principle of the Identity of Automorphic Indiscernibles.**
   T-automorphically absolutely and relationally indiscernible entities are identical.
3. **Isomorphism Representation Thesis.**
   If Repr($\mathfrak{S}$, actual entity), then $\forall \mathfrak{S}' \simeq \mathfrak{S}$: Repr($\mathfrak{S}'$, actual entity).
4. **Structural Explanation.**
   Only structures are needed to explain and to save the phenomena.

- **Quantum Mechanics** (QM) is characterised through its structures.

  A **QM-structure** $\mathfrak{Q}$ standardly set-theoretically defined:

  $$\mathfrak{Q} \equiv \langle \mathcal{S}, \mathcal{H}, \mathcal{O}(\mathcal{H}), \mathcal{U}(\mathcal{H}), \text{Probs} \rangle .$$

- The symmetry groups are:

  $$\text{Gal}(\mathcal{H}) \quad \text{and} \quad \Pi(\mathcal{S}) .$$

Structuralism about QM

0. **Principle of Characterisation of QM**

   QM is characterised through the set of QM-structures $\mathfrak{Q}$.

1. **Principle of Automorphic Properties and Relations**

   $\mathsf{Prop}(\mathcal{S}) \subseteq \mathsf{Aut}(\mathcal{S})$ and $\mathsf{Rel}(\mathcal{S}) \subseteq \mathsf{Aut}(\mathcal{S} \times \mathcal{S})$.

2. **Principle of the Identity of Automorphic (Absolute and Relational) Indiscernibles (PII*)**:

   $\forall\, \boldsymbol{a}, \boldsymbol{b} \in \mathcal{S} \left(\mathsf{AutAbsInd}(\boldsymbol{a}, \boldsymbol{b}) \;\wedge\; \mathsf{AutRelInd}(\boldsymbol{a}, \boldsymbol{b}) \;\longrightarrow\; \boldsymbol{a} = \boldsymbol{b} \right)$.

3. **Structuralist Representation Thesis**

   If Repr($\mathfrak{Q}$, actual physical system),
   then $\forall\, \mathfrak{Q}' \simeq \mathfrak{Q}$: Repr($\mathfrak{Q}'$, actual physical system).

Thesis: Similar elementary particles are entirely indiscernible.

Erwin Schrödinger (Lecture in Dublin, February 1950):

> I beg to emphasize this and I beg you to believe it: it is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of the 'sameness', of identity, really and truly has no meaning.

Received view

Leibniz's Principle of the Identity of Indiscernibles (PII) is refuted by quantum mechanics (QM).

Propounded by:

Schrödinger, Margenau, Cortes, Di Francia, Dalla Chiara, French, Redhead, Teller, French, Krause, Mittelstaedt, Massimi, Castellani, Huggett, Butterfield, . . .

Indiscernibility Thesis (IT):

Similar elementary particles of every kind (fermions, bosons, quons, quarticles, ...) forming composite systems, in all their physical states, pure and mixed, in infinite- and finite-dimensional Hilbert-spaces, are indiscernible.

Metaphysical Discovery: IT is wrong

Similar elementary particles forming a composite system are *categorical, weak relationals*: absolutely indiscernible, i.e. by some property, but weakly discernible, i.e. by an irreflexive, symmetric relation, without appeal to probability-measures.

Set $\mathcal{S}$ of objects; object-variables: $\boldsymbol{a}, \boldsymbol{b} \in \mathcal{S}$.

- ***a*** is ***absolutely discernible***, or an *individual*, iff ***a*** has some physical property that all others lack.
- ***a*** is ***relationally discernible*** iff there is a relation that does not relate ***a*** to all others in the same way.
- ***a*** is ***weakly discernible*** iff ***a*** is relationally discernible by a relation that is irreflexive and symmetric.
- ***a*** is ***relatively discernible*** iff ***a*** is relationally discernible by a relation that is not symmetric.
- ***a*** is ***indiscernible*** iff ***a*** is both absolutely and relationally indiscernible.
- ***a*** is ***discernible*** iff ***a*** is absolutely or relationally discernible, or both.
- ***a*** is ***categorically discernible*** iff ***a*** can be discerned without probabilities.

Leibniz's Principle of the Identity of Indiscernibles (PII) in QM

If physical objects ***a*** and ***b*** are quantum-mechanically indiscernible, then they are identical:

$$\text{PhysInd}(\boldsymbol{a}, \boldsymbol{b}) \longrightarrow \boldsymbol{a} = \boldsymbol{b}\ ,$$

or what is logically the same: distinct physical objects are quantum-mechanically discernible:

$$\boldsymbol{a} \neq \boldsymbol{b} \longrightarrow \neg\text{PhysInd}(\boldsymbol{a}, \boldsymbol{b})\ .$$

PII and IT are each other's negation:

$$\vdash \text{PII} \longleftrightarrow \neg\text{IT}\ .$$

Claim:

$$\text{QM}^{-} \vdash \text{PII} \wedge \neg\text{IT}\ .$$

QM$^-$ characterised by:

- weak magnitude postulate (magnitudes correspond to operators)
- weak property postulate (if eigenstate, then property)
- state postulate
- symmetrisation postulate
- semantic condition

Semantic Condition

Every physical system possesses at every instant in time **at most one** quantitative property $\langle A, a\rangle$ associated with physical magnitude $\mathcal{A}$, where A is an operator and a a number.

so QM^- does not have:

- projection postulate
- probability postulate
- Schrödinger equation
- strong property postulate (eigenstate iff property)

Why not include these?

Not needed for the proofs, save the strong property postulate for the *finite*-dimensional case.

Questions

What is permitted to discern? What is forbidden to discern?

Answer

Only those properties and relations are permitted to occur in PhysInd($\boldsymbol{a}, \boldsymbol{b}$) that meet the following two requirements.

- (Req1) *Physical meaning*.
  All properties and relations should be defined in terms of physical states and physical magnitudes.
  Q: When is an operator physically meaningful?
  A: When it is used by some physicist in some published paper to save some phenomenon.

- (Req2) *Permutation invariance*.
  Every property of one particle is a property of any other; relations should be permutation-invariant, so *binary* relations should be symmetric *and* either reflexive or irreflexive.

Discerning in infinite-dim. Hilbert-space

Composite physical system of $N \geqslant 2$ similar particles.
Associated direct-product Hilbert-space $\mathcal{H}^N = \mathcal{H} \otimes \ldots \otimes \mathcal{H}$.

Lemma 1 (QM^-)

Given N particles. If there are two single-particle operators, A and B, acting in single-particle Hilbert-space $\mathcal{H}$, and they correspond to physical magnitudes $\mathcal{A}$ and $\mathcal{B}$, respectively, and there is a non-zero number $c \in \mathbb{C}$ such that in every pure state $|\phi\rangle \in \mathcal{H}$ in the domain of their commutator the following holds:

$$[A, B]|\phi\rangle = c|\phi\rangle ,$$

then all N particles are categorically weakly discernible.

Proof sketch

Let $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{j}$ be particle-variables, ranging over the set $\{\boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{N}\}$ of $N \geqslant 2$ particles.

- Case for $N = 2$, pure states.

Define the following operators on $\mathcal{H}^2 = \mathcal{H} \otimes \mathcal{H}$:

$$A_{\boldsymbol{1}} \equiv A \otimes \mathbb{1} \quad \text{and} \quad A_{\boldsymbol{2}} \equiv \mathbb{1} \otimes A \,,$$

and *mutatis mutandis* for B.

Define next the following *commutator-relation*:

$$\mathsf{C}(\boldsymbol{a}, \boldsymbol{b}) \quad \text{iff} \quad \forall\, |\Psi\rangle \in \mathcal{D} : \; \big[A_{\boldsymbol{a}}, B_{\boldsymbol{b}}\big]|\Psi\rangle = c|\Psi\rangle \,,$$

where $\mathcal{D} \subseteq \mathcal{H} \otimes \mathcal{H}$ is the domain of the commutator.

Proof sketch (continuation)

- The composite system possesses the following *four* quantitative physical properties (when substituting ***1*** or ***2*** for ***a*** in the first, and ***1*** for ***a*** and ***2*** for ***b***, or conversely, in the second):

$$\langle [A_{\boldsymbol{a}}, B_{\boldsymbol{a}}], c \rangle \quad \text{and} \quad \langle [A_{\boldsymbol{a}}, B_{\boldsymbol{b}}], 0 \rangle \quad (\boldsymbol{a} \neq \boldsymbol{b}) \,.$$

- By Semantic Condition it does not possess the properties

$$\langle [A_{\boldsymbol{a}}, B_{\boldsymbol{a}}], 0 \rangle \quad \text{and} \quad \langle [A_{\boldsymbol{a}}, B_{\boldsymbol{b}}], c \rangle \quad (\boldsymbol{a} \neq \boldsymbol{b}) \,.$$

Hence relation C is symmetric (Req2).

Since by assumption A and B correspond to physical magnitudes, relation C is physically meaningful (Req1).

Thus, since relation C is reflexive and symmetric yet fails for $\boldsymbol{a} \neq \boldsymbol{b}$, and probabilities do not occur in the definition of C, C discerns the two particles *weakly* and *categorically* in every pure state $|\Psi\rangle$ of the composite system.

Proof sketch (continuation)

- Case for $N = 2$, mixed states.

Relation C is easily extended to mixed states $W \in \mathcal{S}(\mathcal{H} \otimes \mathcal{H})$, by

$\mathsf{C}(\boldsymbol{a}, \boldsymbol{b})$ iff $\forall W \in \mathcal{S}(\mathcal{D}) : \left[A_{\boldsymbol{a}}, B_{\boldsymbol{b}}\right] W = cW$,and the ensuing relation also discerns the particles categorically and weakly.

- Case for $N > 2$, mixed and pure states.

Results immediately extended to the N-particle cases, by considering the following N-factor operators:

$$A_{\boldsymbol{j}} \equiv \mathit{1} \otimes \cdots \otimes \mathit{1} \otimes A \otimes \mathit{1} \otimes \cdots \otimes \mathit{1} \ .$$

Q.e.d.

Theorem 1 (QM$^-$).
In a composite physical system of a finite number of similar particles, all particles are categorically weakly discernible in every physical state, pure and mixed, for every infinite-dimensional Hilbert-space.

Proof:
In **Lemma 1**, choose
for A: linear momentum-operator $\widehat{P}$;
for B: Cartesian position-operator $\widehat{Q}$;
for c: $-\mathrm{i}\hbar$.
The physical significance of commutator relation C is grounded in that of $\widehat{P}$ and $\widehat{Q}$ and their commutator:

$$\left[\widehat{P}, \widehat{Q}\right] = -\mathrm{i}\hbar\mathbf{1} \ .$$

$\widehat{P}$ and $\widehat{Q}$ act on (dense subsets of) $L^2(\mathbb{R}^3)$, which is isomorphic to *every infinite-dimensional* Hilbert-space.
Q.e.d.

Including spin

Corollary of **Theorem 1**: result holds for arbitrary spin, not just for spin $s = 0$.

Corrolary 1 (QM$^-$).

In a composite physical system of $N \geqslant 2$ similar particles of arbitrary spin, all particles are categorically weakly discernible in every admissible physical state, pure and mixed, for every infinite-dimensional Hilbert-space.

Proof:
Reformulation of Proof of **Theorem 1**, starting with spinorial wave-functions (for spin $s = n\hbar/2$ and $s = (n+1)\hbar/2$):

$$\mathcal{H}_s \equiv \left(L^2(\mathbb{R}^3)\right)^{2s+1} .$$

Q.e.d.

Remarks

1. Algebraic hall-mark of QM^- used in the proofs.
   Non-commutativity known since the advent of QM!
   What we didn't know, but do know now, is that this old knowledge provides the ground for discerning similar particles weakly and categorically.
2. Two bosons in symmetric direct-product states, say
   $$\Psi(\boldsymbol{q}_1, \boldsymbol{q}_2) = \phi(\boldsymbol{q}_1)\phi(\boldsymbol{q}_2) ,$$
   are also weakly discernible.
3. Minimal use of symmetrisation postulate.
4. Every 'realistic' quantum-mechanical model of a physical system, whether in atomic physics, nuclear physics or solid-state physics, employs wave-functions. *Now*, *and only now*, we can conclude that the similar elementary particles of the real world are categorically and weakly discernible.

Ontological Lesson that QM is teaching us:

Similar elementary particles forming a composite physical system are categorical, weak discernibles.

$$\text{QM}^- \vdash \forall \boldsymbol{a}, \boldsymbol{b} \in \mathcal{S}_N : \ \boldsymbol{a} \neq \boldsymbol{b} \ \longrightarrow \ \neg\text{PhysInd}(\boldsymbol{a}, \boldsymbol{b}) \ ,$$

which is logically the same as having proved PII:

$$\text{QM}^- \vdash \forall \boldsymbol{a}, \boldsymbol{b} \in \mathcal{S}_N : \ \text{PhysInd}(\boldsymbol{a}, \boldsymbol{b}) \ \longrightarrow \ \boldsymbol{a} = \boldsymbol{b} \ ,$$

and by the earlier theorem of logic as having disproved IT. Hence

$$\text{QM}^- \vdash \ \text{PII} \ \wedge \ \neg\text{IT} \ .$$

Conclusions and Remarks concerning QM

1. **Structuralism** is realist in the sense that it is committed to the existence of elementary particles, but their existence as physical discernibles relies on their being members of the domain of QM-**structure** $\mathfrak{Q}$ and relies on that very structure $\mathfrak{Q}$ generating physically significant and automorphic relations that discern them.

2. Structuralist objects, if they must exist, should be relationals. That is exactly what elementary particles demonstrably are: relationals. This is one version of **Ontic Structuralism**.

3. No **haecceity** needed: the **Principle of the Identity of Indiscernibles** (PII) demonstrably holds in QM.

- **General Theory of Relativity** (GTR) is characterised through its structures.

  A **GTR-structure** $\mathfrak{G}$ standardly set-theoretically defined:

  $$\mathfrak{G} \equiv \langle \mathcal{M}, \mathcal{T}(\mathcal{M}), \mathcal{C}(\mathcal{M}), \mathcal{A}(\mathcal{M}), g, T \rangle ,$$

  where g and T are related by Einstein's Field Equations.

  Relevant composition-groups of one-one mappings $\mathcal{M} \to \mathcal{M}$:

  $$\text{Aut}(\mathfrak{G}) \subseteq \text{Sym}(\mathfrak{G}) \subseteq \text{Caus}(\mathfrak{G}) \subseteq \text{Diff}(\mathfrak{G}) \subseteq \text{Hom}(\mathfrak{G}) \subset \Pi(\mathcal{M}) .$$

- **Special Theory of Relativity** (STR) has a fixed space-time structure,

  the **STR-structure**:

  $$\mathfrak{M} \equiv \langle \mathcal{M}, \mathcal{T}_{\text{Zmn}}(\mathcal{M}), \mathcal{C}(\mathcal{M}), \mathcal{A}(\mathcal{M}), \eta, 0 \rangle .$$

Structuralism about GTR

0. Principle of Characterisation of GTR

GTR is characterised through the set of GTR-structures $\mathfrak{G}$.

1. Principle of Automorphic Properties and Relations

$\text{Prop}(\mathcal{M}) \subseteq \text{Aut}(\mathcal{M})$ and $\text{Rel}(\mathcal{M}) \subseteq \text{Aut}(\mathcal{M} \times \mathcal{M})$.

2. Principle of the Identity of Automorphic (Absolute and Relational) Indiscernibles (PII*):

$\forall\, p, q \in \mathcal{M} \left(\text{AutAbsInd}(p,q) \ \wedge \ \text{AutRelInd}(p,q) \ \longrightarrow \ p = q \right)$.

3. Structuralist Representation Thesis

If Repr($\mathfrak{G}$, (part of) the universe),
then $\forall\, \mathfrak{G}' \simeq \mathfrak{G}$: Repr($\mathfrak{G}'$, (part of) the universe).

Recall:

Definitions

Absolute Indiscernibility for $p, q \in \mathcal{M}$:

$\text{AbsInd}(p, q, \mathcal{M})$ iff $\forall P \in \text{Prop}(\mathcal{M}) : \; p \in P \longleftrightarrow q \in P$.

Relational Indiscernibility for $p, q \in \mathcal{M}$:

$\text{RelInd}(p, q, \mathcal{M})$ iff $\forall R \in \text{Rel}(\mathcal{M})$:

$$\big(\forall r \in \mathcal{M} : \; \langle p, r\rangle \in R \longleftrightarrow \langle q, r\rangle \in R\big) \; \wedge$$
$$\big(\forall s \in \mathcal{M} : \; \langle s, p\rangle \in R \longleftrightarrow \langle s, q\rangle \in R\big) \, .$$

- Every 3-dimensional space Σ_t of a cosmic space-time structure is a Riemannian differentiable manifold and therefore carries a *distance function*:

  $$d_t : \Sigma_t \times \Sigma_t \to \mathbb{R}, \ \ \langle p, q \rangle \mapsto d_t(p,q) \ .$$

Definition

Let $\mathcal{S}$ be any set. Function $d : \mathcal{S} \times \mathcal{S} \to \mathbb{R}$ is a **distance function** iff

d meets the Frèchet conditions:

(F1) $d(p,q) = d(q,p)$,

(F2) $d(p,q) + d(q,r) \geqslant d(p,r)$,

(F3) $p = q \ \longrightarrow \ d(p,q) = 0$ and $p \neq q \longrightarrow d(p,q) > 0$.

- A distance function d yields an **identity criterion** for the members of $\mathcal{S}$:

$$p = q \quad \longleftrightarrow \quad d(p,q) = 0\,.$$

Thus **Distance relation** discerns members of $\mathcal{S}$ relationally:

$$D(p,q) \quad \text{iff} \quad d(p,q) > 0\,,$$

because then:

$$\neg D(p,q) \quad \longleftrightarrow \quad p = q \quad \text{and} \quad D(p,q) \quad \longleftrightarrow \quad p \neq q\,.$$

- **Remark**

Relation D discerns the points symmetrically and irreflexively, therefore **weakly**; space-points belong to the metaphysical category of **relationals** (neither individuals nor indiscernibles).

■ Construction of a spatial distance function d_t in Σ_t

In local co-ordinate chart $\langle F, O\rangle \in \mathcal{A}(\Sigma_t)$, we have:

$$dr^2 = \sum_{j,k=1}^{3} h_{jk}(t) dx_j dx_k ,$$

where F sends $p \in O$ to $\mathbf{x}(p) \in \mathbb{R}^3$. Then

$$d_F(p,q) \equiv \int_{\mathbf{x}(p)}^{\mathbf{x}(q)} dr$$

where one integrates along the straightest path (geodesic) in Σ_t.

This d_F obeys the Frèchet conditions (F1)–(F3).

Remark

Let F be a Cartesian co-ordinate chart $p \mapsto \langle t_p, x_p, y_p, z_p \rangle$ on covering region $O \in \mathcal{C}(\mathcal{M})$, assumed approximately Minkowskian.
The spatio-temporal function

$$s(p,q) \;\equiv\; \sqrt{\big(c^2(t_p - t_q)^2 - (x_p - x_q)^2 - (y_p - y_q)^2 - (z_p - z_q)^2\big)}\;.$$

violates Frèchet conditions (F2) and (F3): **not** a distance function that generates a distance relation to discern points relationally in $O \subset \mathcal{M}$.

But in every 3-dimensional simultaneity space $F_t[O]$ the Euclidean distance relation d_{E} does discern points relationally.
Define:

$$Q_F^{\mathsf{Mink}}(p,q) \;\text{ iff }\; ct_p \neq ct_q \;\vee\; d_{\mathsf{E}}(p,q) > 0\;.$$

- Consider frame $F[O]$, for an arbitrary $O \in \mathcal{C}(\mathcal{M})$.
  On 3-dim. $F_t[O]$, restriction h of g yields infinitesimal line-element dr.
  Then again, in frame $F_t[O]$:

$$d_F(p,q) \;\equiv\; \int_{\mathbf{x}(p)}^{\mathbf{x}(q)} dr$$

  Frame relation:

$$Q_F(p,q) \;\text{ iff }\; ct_p \neq ct_q \;\vee\; d_F(p,q) > 0$$

  discerns $p, q \in O$ relationally:

$$\neg Q_F(p,q) \;\longleftrightarrow\; p = q \,.$$

- **Fact.** If Q_F discerns p and q, then so does $Q_{F'}$, for every other co-ordinate chart $F' \in \mathcal{A}_O(\mathcal{M})$.

Then for every $p, q \in \mathcal{M}$, we define the following discerning relation:

$$Q(p,q) \text{ iff } \exists\, O \in \mathcal{C}(\mathcal{M}) \big(p, q \in O \ \wedge\ \forall\, \langle F, O \rangle \in \mathcal{A}_O(\mathcal{M}) :\ Q_F(p,q) \big) .$$

- But what if $p \in O$ and $q \in O'$ do not lie in **one** covering region?

  Chain of overlapping open covers. Abbreviations:

  Geod$(\lambda;\, p, q)$ iff λ is a geodesic in $\mathcal{M}$ with end-points p and q.

  $Q(O;\, p, q)$ iff $\forall\, \langle F, O \rangle \in \mathcal{A}_O(\mathcal{M}) :\ Q_F(p,q)$.

Definition

$S(p,q)$ iff

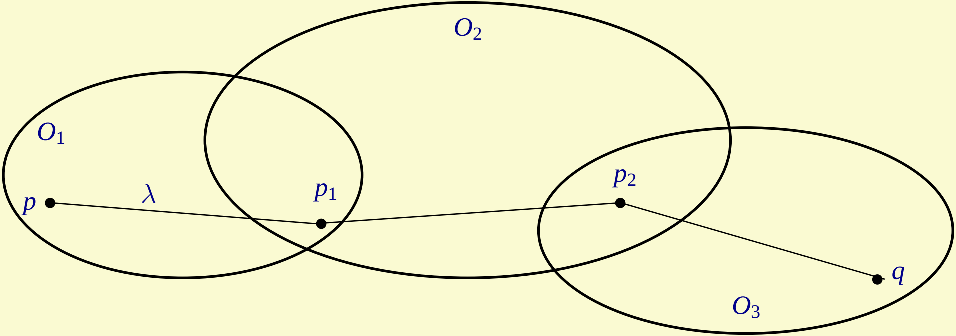
$$\exists\, n \in \mathbb{N}^+,\ \exists\, O_1, \ldots, O_n \in \mathcal{C}(\mathcal{M}),\ \exists\, \lambda \subset \mathcal{M},\ \exists\, p_1 \in (O_1 \cap O_2 \cap \lambda),$$

$$\exists\, p_2 \in (O_2 \cap O_3 \cap \lambda),\ \ldots,\ \exists\, p_{n-1} \in (O_{n-1} \cap O_n \cap \lambda) :$$

$$\text{Geod}(\lambda;\, p, q) \ \wedge\ \lambda \subset \textstyle\bigcup_{j=1}^{n} O_j \ \wedge\ p \in O_1 \backslash O_2 \ \wedge\ q \in O_n \backslash O_{n-1}$$

$$\wedge\ Q(O_1;\, p, p_1) \ \wedge\ Q(O_2; p_1, p_2) \ \wedge\ \ldots\ \wedge\ Q(O_n;\, p_{n-1}, q) .$$

Case of $\neg S(p, q)$ for $n = 3$:



Theorem.

Space-time points in every open, connected generally-relativistic space-time are automorphically and weakly discernible by a metric-based relation; therefore absolutely indiscernible space-time points are relationals.

Proof proceeds by demonstrating that relation $S(p, q)$ discerns the points automorphically and provides an **identity criterion**:

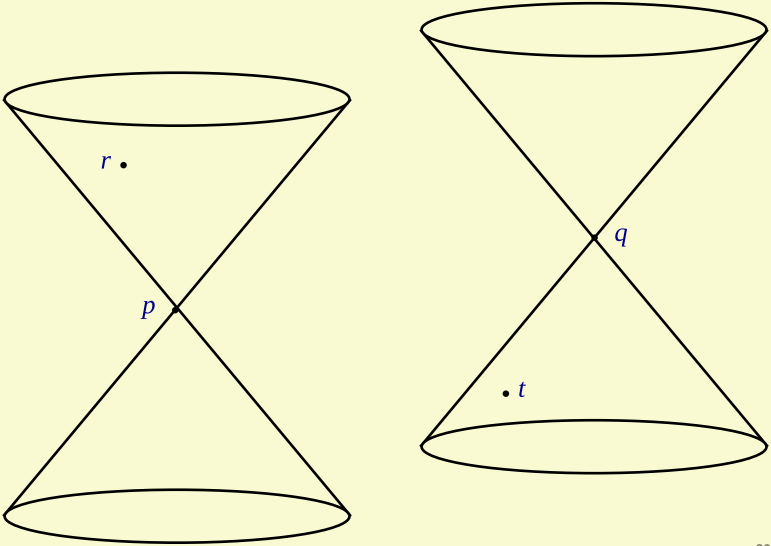
$$\neg S(p, q) \ \longleftrightarrow \ p = q \, .$$

Theorem.

Space-time points in every open, connected generally-relativistic space-time are conformally and weakly discernible by a light-cone relation.

Proof is based on a **light-cone relation**, L, that relates points iff there is some point inside one light-cone but outside the other light-cone of these points:

$L(p,q)$ iff $\big(\exists\, r \in \mathcal{M} : r \in LC(p) \setminus LC(q)\big) \vee \big(\exists\, t \in \mathcal{M} : t \in LC(q) \setminus LC(p)\big)$.



■ Theorem

Space-time points in every open, connected generally-relativistic space-time structure are homeomorphically and weakly discernible.

Proof considers the **curve-relation**:

$C(p,q)$ iff p and q can be connected by an open curve in $\mathcal{M}$,

and demonstrates that C yields an identity criterion:

$$\neg C(p,q) \quad \longleftrightarrow \quad p = q \ ,$$

and discerns points homeomorphically.

Conclusions and Remarks concerning GTR and STR

0. A variety of automorphic and physically significant relations can be found to discern space-time points in most GTR-space-times; they all yield identity criteria.

1. **Structuralism** is realist in the sense that it is committed to the existence of space-time points, but their existence as physical discernibles relies on their being members of the domain $\mathcal{M}$ of GTR-**structure** $\mathfrak{G}$ and relies on that very structure $\mathfrak{G}$ generating physically significant and automorphic relations that discern them; the points are not individuals (but relationals) because generically not absolutely discernible, and therefore they cannot be individuated (but they can be discerned).

2. **Structuralism** obviously is not relationism, if only because of **1**.

3. **Structuralism** is also not substantivalism when substantivalism is committed to the rejection of Leibniz Equivalence, because that entails a rejection of the **Structuralist Representation Thesis** — not even sophisticated varieties e.g. O. Pooley (2006: 99–103). Notably, **structuralism** does not fall prey to the Hole Argument.

4. No **haecceity** needed: the **Principle of the Identity of Automorphic Indiscernibles** (PII*) demonstrably holds in GTR and STR.

5. No **possible worlds** needed — **Structuralist Representation Thesis** only speaks of the universe (‘the actual world’).

6. Impression: **Structuralism** is the view on space-time that fits the views of the working physicists and mathematical physicists best.

Substantivalism–Relationism Debate about Nature of Space-Time

J.E. Earman in *World Enough and Space-Time* (1989, p. 208):

> My own tentative conclusion from this unsatisfactory situation is that when the smoke of the battle finally clears, what will emerge is a conception of space-time that fits neither traditional relationism nor substantivalism. At present we can see only dimly if at all the outlines the third alternative might take. But I hope to have identified the considerations we need to pursue in trying to give it a more definite form.

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Space-time structuralism IS that tertium quid.

Dorato (2000), Dorato & Pauri (2006), Saunders (2002; 2003), Bain (2006), Rickles & French (2006), Pooley (2006), Stachel (2006), Ladyman & Ross (2007: 141–145), Esfeld & Lam (2008), . . .

ALAS!

No time.

Next time, if there be a next time ...

ALAS!

No time.

Next time, if there be a next time ...

Please don't forget (likely to happen up North, here in Groningen):

Structuralism is a sizzlingly exciting and volcanically hot philosophical research programme at the cross roads of philosophy of science, of physics, of mathematics, of logic, and of analytic metaphysics to work on!

THIS is where philosophical action is!

The Argument from Quantum Field Theory (QFT)

S1. A quantum-theory of particles in a Minkowskian space-time is mathematically impossible (Malament's Theorem).

S2. The Standard Model is formulated in the framework of Relativistic Quantum Field Theory (RQFT), which is a quantum-theory on Minkowskian space-time. The ontological substances of RQFT are **space-time** and the **quantum field**.

S3. The Standard Model of (non-gravitationally) interacting matter is not a particle theory (from S2) and cannot be interpreted as a particle-theory (from S1, S2).

S4. The vacuum of RQFT is **unlike** literally empty space-time, i.e. space-time without material particles (Casimir effect, Unruh effect, Reeh-Schlieder Theorem); it is a plenum seething with activity.

S5. The Standard Model currently is the best scientific theory of (non-gravitationally) interacting matter.

S6. The currently best scientific theory of matter and its interactions (save gravity) is not a particle-theory (from S4, S5).

S7. The currently best guess of the fundamental ontological substance of the universe is the **relativistic quantum field** (from S2, S6).

S8. The words ‘quantum field’ express first and foremost a **mathematical** concept.

S9. Setting the Teller-Maxwell interpretation of the quantum field as a ‘propensity-field’ aside (because . . .), there is no way known of how to interpret — let alone how to depict — the mathematical concept of a quantum field **physically**; we only know how to use this **mathematical concept** to calculate cross-sections and other measurables.

S10. Thus consider the relativistic quantum field as a *sui generis* **physical structure** (in the light of S7, S8, S9).

S11. The universe consists of physical structures (from S4, S7, S10). This is **Ontic Structuralism: the world consists of structure(s)**.