Discerning Elementary Particles

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Talk based on:

- F.A. Muller & M.P. Seevinck, 'Discerning Elementary Particles', *Philosophy of Science* **75** (2009)
- F.A. Muller, 'The Rise of Relationals', submitted to *Mind*

Opening

Thesis: Similar elementary particles are entirely indiscernible.

Erwin Schrödinger (Lecture in Dublin, February 1950):

> *I beg to emphasize this and I beg you to believe it: it is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of the 'sameness', of identity, really and truly has no meaning.*

Received view

Leibniz's Principle of the Identity of Indiscernibles (PII) is refuted by quantum mechanics (QM).

Propounded by:

Schrödinger, Margenau, Cortes, Di Francia, Dalla Chiara, French, Redhead, Teller, French, Krause, Mittelstaedt, Massimi, Castellani, Huggett, Butterfield, . . .

Opening

Indiscernibility Thesis (IT):

Similar elementary particles of every kind (fermions, bosons, quons, quarticles, . . .) forming composite systems, in all their physical states, pure and mixed, in infinite- and finite-dimensional Hilbert-spaces, are indiscernible.

Metaphysical Discovery: IT is wrong

Similar elementary particles forming a composite system are *categorical, weak relationals*: absolutely indiscernible, i.e. by some property, but weakly discernible, i.e. by an irreflexive, symmetric relation, without appeal to probability-measures.

Completion of the project initiated and developed in:

- Muller & Saunders, 'Discerning Fermions', *British Journal for the Philosophy of Science* **59** (2008) 499–548
- Saunders, 'Are Quantum Particles Objects?', *Analysis* **66.1** (2006) 52–63.

Outline

1. Preliminaries
2. Discerning in infinite-dimensional Hilbert-space
3. Discerning in finite-dimensional Hilbert-space
4. The Circularity Objection
5. Conclusions

Section 1. Preliminaries

Set $\mathcal{S}$ of objects; object-variables: $\boldsymbol{a}, \boldsymbol{b} \in \mathcal{S}$.

- ***a*** is ***absolutely discernible***, or an *individual*, iff ***a*** has some physical property that all others lack.
- ***a*** is ***relationally discernible*** iff there is a relation that does not relate ***a*** to all others in the same way.
- ***a*** is ***weakly discernible*** iff ***a*** is relationally discernible by a relation that is irreflexive and symmetric.
- ***a*** is ***relatively discernible*** iff ***a*** is relationally discernible by a relation that is not symmetric.
- ***a*** is ***indiscernible*** iff ***a*** is both absolutely and relationally indiscernible.
- ***a*** is ***discernible*** iff ***a*** is absolutely or relationally discernible, or both.

Leibniz's Principle of the Identity of Indiscernibles (PII) in QM

If physical objects $\boldsymbol{a}$ and $\boldsymbol{b}$ are quantum-mechanically indiscernible, then they are identical:

$$\text{PhysInd}(\boldsymbol{a}, \ \boldsymbol{b}) \ \longrightarrow \ \boldsymbol{a} = \boldsymbol{b} \ ,$$

or what is logically the same: distinct physical objects are quantum-mechanically discernible:

$$\boldsymbol{a} \neq \boldsymbol{b} \ \longrightarrow \ \neg\text{PhysInd}(\boldsymbol{a}, \ \boldsymbol{b}) \ .$$

PII and IT are each other's negation:

$$\vdash \ \text{PII} \ \longleftrightarrow \ \neg\text{IT} \ .$$

Claim:

$$\text{QM}^{-} \ \vdash \ \text{PII} \ \wedge \ \neg\text{IT} \ .$$

QM^{-} characterised by:

- weak property postulate (if eigenstate, then property)
- weak magnitude postulate (magnitudes correspond to operators)
- state postulate
- symmetrisation postulate
- semantic condition

Semantic Condition

Every physical system possesses at every instant in time **at most one** quantitative property $\langle A, a \rangle$ associated with physical magnitude $\mathcal{A}$, where A is an operator and a a number.

so QM^{-} does not have:

- projection postulate
- probability postulate
- Schrödinger equation
- strong property postulate (eigenstate iff property)

Why not include these?

Not needed for the proofs, save the strong property postulate for the *finite*-dimensional case.

Section 1. License to Discern

Questions

What is permitted to discern? What is forbidden to discern?

Answer

Only those properties and relations are permitted to occur in PhysInd$(\boldsymbol{a}, \boldsymbol{b})$ that meet the following two requirements.

- (Req1) *Physical meaning*.
  All properties and relations should be defined in terms of physical states and physical magnitudes.
  Q: When is an operator physically meaningful?
  A: When it is used by some physicist in some published paper to save some phenomenon.

- (Req2) *Permutation invariance*.
  Every property of one particle is a property of any other; relations should be permutation-invariant, so *binary* relations should be symmetric *and* either reflexive or irreflexive.

Composite physical system of $N \geqslant 2$ similar particles.
Associated direct-product Hilbert-space $\mathcal{H}^N = \mathcal{H} \otimes \ldots \otimes \mathcal{H}$.

Lemma 1 (QM$^-$)

Given N particles. If there are two single-particle operators, A and B, acting in single-particle Hilbert-space $\mathcal{H}$, and they correspond to physical magnitudes $\mathcal{A}$ and $\mathcal{B}$, respectively, and there is a non-zero number $c \in \mathbb{C}$ such that in every pure state $|\phi\rangle \in \mathcal{H}$ in the domain of their commutator the following holds:

$$[A, B]|\phi\rangle = c|\phi\rangle ,$$

then all N particles are categorically weakly discernible.

Proof sketch

Let $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{j}$ be particle-variables, ranging over the set $\{\boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{N}\}$ of $N \geqslant 2$ particles.

- Case for $N = 2$, pure states.

Define the following operators on $\mathcal{H}^2 = \mathcal{H} \otimes \mathcal{H}$:

$$A_{\boldsymbol{1}} \equiv A \otimes \boldsymbol{1} \quad \text{and} \quad A_{\boldsymbol{2}} \equiv \boldsymbol{1} \otimes A \,,$$

and *mutatis mutandis* for B.

Define next the following *commutator-relation*:

$$\mathrm{C}(\boldsymbol{a}, \boldsymbol{b}) \quad \text{iff} \quad \forall\, |\Psi\rangle \in \mathcal{D} : \ \left[A_{\boldsymbol{a}}, B_{\boldsymbol{b}}\right] |\Psi\rangle \ = \ c|\Psi\rangle \ ,$$

where $\mathcal{D} \subseteq \mathcal{H} \otimes \mathcal{H}$ is the domain of the commutator.

Proof sketch (continuation)

- The composite system possesses the following *four* quantitative physical properties (when substituting ***1*** or ***2*** for ***a*** in the first, and ***1*** for ***a*** and ***2*** for ***b***, or conversely, in the second):

$$\left\langle \left[A_{\boldsymbol{a}},\, B_{\boldsymbol{a}}\right],\, c \right\rangle \quad \text{and} \quad \left\langle \left[A_{\boldsymbol{a}},\, B_{\boldsymbol{b}}\right],\, 0 \right\rangle \quad (\boldsymbol{a} \neq \boldsymbol{b})\ .$$

- By Semantic Condition it does not possess the properties

$$\left\langle \left[A_{\boldsymbol{a}},\, B_{\boldsymbol{a}}\right],\, 0 \right\rangle \quad \text{and} \quad \left\langle \left[A_{\boldsymbol{a}},\, B_{\boldsymbol{b}}\right],\, c \right\rangle \quad (\boldsymbol{a} \neq \boldsymbol{b})\ .$$

Hence relation C is symmetric (Req2).

Since by assumption A and B correspond to physical magnitudes, relation C is physically meaningful (Req1).

Thus, since relation C is reflexive and symmetric yet fails for $\boldsymbol{a} \neq \boldsymbol{b}$, and probabilities do not occur in the definition of C, C discerns the two particles *weakly* and *categorically* in every pure state $|\Psi\rangle$ of the composite system.

Proof sketch (continuation)

- Case for $N = 2$, mixed states.

Relation C is easily extended to mixed states $W \in \mathcal{S}(\mathcal{H} \otimes \mathcal{H})$, by

$$\mathsf{C}(\boldsymbol{a}, \boldsymbol{b}) \quad \text{iff} \quad \forall\, W \in \mathcal{S}(\mathcal{D}) : \ \left[A_{\boldsymbol{a}}, B_{\boldsymbol{b}}\right] W \ = \ cW \ ,$$

and the ensuing relation also discerns the particles categorically and weakly.

- Case for $N > 2$, mixed and pure states.

Results immediately extended to the N-particle cases, by considering the following N-factor operators:

$$A_{\boldsymbol{j}} \equiv \mathit{1} \otimes \ \cdots \ \otimes \mathit{1} \otimes A \otimes \mathit{1} \otimes \ \cdots \ \otimes \mathit{1} \ .$$

Q.e.d.

Theorem 1 (QM^{-}).

In a composite physical system of a finite number of similar particles, all particles are categorically weakly discernible in every physical state, pure and mixed, for every infinite-dimensional Hilbert-space.

Proof:

In **Lemma 1**, choose

- for A: linear momentum-operator $\widehat{P}$;
- for B: Cartesian position-operator $\widehat{Q}$;
- for c: $-\mathrm{i}\hbar$.

The physical significance of commutator relation C is grounded in that of $\widehat{P}$ and $\widehat{Q}$ and their commutator:

$$\left[\widehat{P},\, \widehat{Q}\right] = -\mathrm{i}\hbar\mathbb{1} \ .$$

$\widehat{P}$ and $\widehat{Q}$ act on (dense subsets of) $L^2(\mathbb{R}^3)$, which is isomorphic to *every infinite-dimensional* Hilbert-space.

Q.e.d.

Including spin

Corollary of **Theorem 1**: result holds for arbitrary spin, not just for spin $s = 0$.

Corrolary 1 (QM$^-$).

In a composite physical system of $N \geqslant 2$ similar particles of arbitrary spin, all particles are categorically weakly discernible in every admissible physical state, pure and mixed, for every infinite-dimensional Hilbert-space.

Proof:
Reformulation of Proof of **Theorem 1**, starting with spinorial wave-functions (for spin $s = n\hbar/2$ and $s = (n+1)\hbar/2$):

$$\mathcal{H}_s \equiv \left(L^2(\mathbb{R}^3)\right)^{2s+1} .$$

Q.e.d.

Remarks

1. Algebraic hall-mark of QM^{-} used in the proofs.
   Non-commutativity known since the advent of QM!
   What we didn't know, but do know now, is that this old knowledge provides the ground for discerning similar particles weakly and categorically.
2. Two bosons in symmetric direct-product states, say
   $$\Psi(\boldsymbol{q}_1, \boldsymbol{q}_2) = \phi(\boldsymbol{q}_1)\phi(\boldsymbol{q}_2) \; ,$$
   are also weakly discernible.
3. Minimal use of symmetrisation postulate.
4. Every 'realistic' quantum-mechanical model of a physical system, whether in atomic physics, nuclear physics or solid-state physics, employs wave-functions. *Now*, *and only now*, we can conclude that the similar elementary particles of the real world are categorically and weakly discernible.
   - Conjecture 1 of Muller & Saunders (2008: 537) proved.

Theorem 2 (QM$^-$ with Strong Property Postulate).
In a composite physical system of $N \geqslant 2$ similar particles, all particles are categorically weakly discernible in every physical state, pure and mixed, for every finite-dimensional Hilbert-space by only using their spin degrees of freedom.

Proof Sketch.
Core is 'Total-spin relation' (for spin-s particles):

$$\mathsf{T}(\boldsymbol{a}, \boldsymbol{b}) \quad \text{iff} \quad \forall\, |\phi\rangle \in \mathbb{C}^{2s+1} \otimes \mathbb{C}^{2s+1} :$$

$$\left(\widehat{\mathbf{S}}_{\boldsymbol{a}} + \widehat{\mathbf{S}}_{\boldsymbol{b}}\right)^2 |\phi\rangle \;=\; 4s(s+1)\hbar^2\, |\phi\rangle \;,$$

where total spin operator of the composite system is:

$$\widehat{\mathbf{S}} \;\equiv\; \widehat{\mathbf{S}}_{\mathbf{1}} + \widehat{\mathbf{S}}_{\mathbf{2}} \;, \quad \text{where} \quad \widehat{\mathbf{S}}_{\mathbf{1}} \;\equiv\; \widehat{\mathbf{S}} \otimes \mathit{1} \;, \quad \widehat{\mathbf{S}}_{\mathbf{2}} \;\equiv\; \mathit{1} \otimes \widehat{\mathbf{S}} \;,$$

Proof Sketch (continuation)

and z-component is:

$$\widehat{S}_z = \widehat{S}_z \otimes \mathbf{1} + \mathbf{1} \otimes \widehat{S}_z \ ,$$

which all act in $\mathbb{C}^{2s+1} \otimes \mathbb{C}^{2s+1}$.

Set of commuting self-adjoint spin-operators:

$$\left\{\widehat{\mathbf{S}}_{\mathbf{1}},\ \widehat{\mathbf{S}}_{\mathbf{2}},\ \widehat{\mathbf{S}}\ ,\ \widehat{S}_z\right\}$$

Common set of orthonormal eigenvectors $|s; S, M\rangle$. Eigenvector equations:

$$\begin{aligned}
\widehat{\mathbf{S}}_{\mathbf{1}}^{2}|s;\, S, M\rangle &= s(s+1)\hbar^2\, |s;\, S, M\rangle \ , \\
\widehat{\mathbf{S}}_{\mathbf{2}}^{2}|s;\, S, M\rangle &= s(s+1)\hbar^2\, |s;\, S, M\rangle \ , \\
\widehat{\mathbf{S}}^{2}|s;\, S, M\rangle &= S(S+1)\hbar^2\, |s;\, S, M\rangle \ , \\
\widehat{S}_z\, |s;\, S, M\rangle &= M\hbar\, |s;\, S, M\rangle \ .
\end{aligned}$$

Proof Sketch (continuation)

Relation T meets Req1 and Req2 and discerns the two particles weakly.

- *Case 1*: $\boldsymbol{a} = \boldsymbol{b}$.

Composite system possesses following quantitative physical property (when substituting ***1*** or ***2*** for ***a***):

$$\left\langle 4\widehat{\mathbf{S}}_{\boldsymbol{a}}^{2},\, 4s(s+1)\hbar^{2} \right\rangle .$$

This property *is* a relation between the constituent parts of the system, and this relation is reflexive: $\mathrm{T}(\boldsymbol{a}, \boldsymbol{a})$ for every ***a***.

- *Case 2*: $\boldsymbol{a} \neq \boldsymbol{b}$.

According to weak property postulate:

$$\left\langle \left(\widehat{\mathbf{S}}_{\boldsymbol{a}} + \widehat{\mathbf{S}}_{\boldsymbol{b}}\right)^{2},\, S(S+1)\hbar^{2} \right\rangle .$$

Proof Sketch (continuation)

Composite system does *not* possess, by semantic condition, the following *two* quantitative physical properties of the composite system (substitute ***1*** for ***a*** and ***2*** for ***b*** or conversely):

$$\left\langle \left(\widehat{\mathbf{S}}_{\boldsymbol{a}} + \widehat{\mathbf{S}}_{\boldsymbol{b}}\right)^2,\ s(s+1)\hbar^2 \right\rangle ,$$

expressed by predicate T as a relation between its constituent parts, ***1*** and ***2***.

However, for superpositions of basis-vectors having a different value for S, such as

$$\tfrac{1}{\sqrt{2}}\left(|s;\ 0,0\rangle + |s;\ 1,M\rangle\right) ,$$

where M is -1, 0 or $+1$, which are *not* eigenstates of total spin-operator $\widehat{\mathbf{S}}$, strong property postulate needed.

- Extension to mixed states straightforward.

Q.e.d.

Remarks

1. Strong Property Postulate needed,
   gives rise to Projection Postulate.

2. All theorems imply probabilistic versions.
   – Probability Postulate (Born-rule) is then needed
   – Weak Property Postulate and Semantic Condition not needed

Michiel leaves the stage [end of Mini-Symposium *Act 1, Scene 1*],
Fred enters [*Act 1, Scene 2* commences]

Back to Max Black's Case (*Mind*, 1952).

Two black iron spheres, of radius 1 mile, next to each other, called Castor and Pollux, in a further empty universe.

Violation of PII?

Absolutely indiscernible but weakly discerned by relation

$D(x, y)$ iff sphere x is 2 miles apart from sphere y ,

where x and y are sphere-variables ranging over $\{$ Castor, Pollux $\}$.

R. Barcan Marcus, in *Modalities: Philosophical Essays* (1993, pp. 20–25),
S. French & D. Krause, in their *Identity in Physics* (2006, pp. 167–172):

Assuming that Castor $\neq$ Pollux (i.e. there are two spheres) makes proof of numerical diversity circular!

When assuming that Castor $=$ Pollux (i.e. there is a single sphere bearing two names), nothing of the sort using distance-relation $D(\cdot,\cdot)$ can be proved!

Mutatis mutandis for the quantum case.

- Conclusion: proof is totally unconvincing.

Elaboration of the Circularity Objection

Elaboration of circularity charge, and vindicating it, by K. Hawley in 'Identity and Indiscernibility', *Mind* **118** (2009), pp. 102–119, by means of considering properties :

$H_1(x)$ iff sphere x is 2 miles from Castor ;

$H_2(x)$ iff sphere x is 2 miles from Pollux .

Properties can only discern when Castor $\neq$ Pollux, otherwise $H_1(\cdot)$ and $H_2(\cdot)$ the same.

Unless $H_1(\cdot)$ and $H_2(\cdot)$ are more fundamental than spatial relation $D(x, y)$. Implausible.

So one must assume that Castor $\neq$ Pollux.

- Circularity again.

Ground rules for discussing threats to PII

Consider some arrangement potentially threatening PII.

Proposed general 3-step procedure (**I** and **III** by Hawley):

I. A description of the arrangement.

II(a). What does PII meaningfully apply to?
II(b). What sort of features are admitted to discern?
II(c). What sort of features are forbidden to discern?

III. An argument that in this arrangement we have distinct but indiscernible objects.

Circularity Charge Exposed

Attempt to answer to **II(b)** and **II(c)**

Permitted to discern are those properties and relations expressed by predicates that:

- do not contain names, and
- follow from the description of the arrangement (**I**) and therefore do not break the symmetry of the arrangement;
- what is not permitted is forbidden.

What is wrong with the circularity charge?

1. Fallacy of equivocation with respect to what is permitted and what is forbidden.
2. No *petitio principii* because what is proved is not mere numerical diversity but *that there is a permitted relation that discerns the objects weakly*. This is not assumed: it is demonstrated.

Section V. Conclusions

1. Ontological Lesson that QM is teaching us:

Similar elementary particles forming a composite physical system are categorical, weak discernibles.

$$\mathrm{QM}^{-} \vdash \forall \boldsymbol{a}, \boldsymbol{b} \in \mathcal{S}_N : \ \boldsymbol{a} \neq \boldsymbol{b} \ \longrightarrow \ \neg \mathrm{PhysInd}(\boldsymbol{a}, \boldsymbol{b}) \ ,$$

which is logically the same as having proved PII:

$$\mathrm{QM}^{-} \vdash \forall \boldsymbol{a}, \boldsymbol{b} \in \mathcal{S}_N : \ \mathrm{PhysInd}(\boldsymbol{a}, \boldsymbol{b}) \ \longrightarrow \ \boldsymbol{a} = \boldsymbol{b} \ ,$$

and by the earlier theorem of logic as having disproved IT. Hence

$$\mathrm{QM}^{-} \vdash \ \mathrm{PII} \ \wedge \ \neg \mathrm{IT} \ .$$

2. *The circularity charge is unconvincing.*

Fred leaves the stage [end of Mini-Symposium *Act 1*]

Adam enters [*Act 2* commences]