Analyzing passion at a distance: progress in experimental metaphysics?

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Prospects & Introduction

I will consider the well-known EPR-Bohm setup to test Bell's inequality.

Question: What kind of information—about the distant measurement setting or the outcome or both—and which amount of it has to be non-locally available to simulate the violation of the Clauser–Horne–Shimony–Holt (CHSH) inequality within the framework of non-local realistic models?

To be shown: it is impossible to model a violation without having information in one laboratory about *both* the setting and the outcome at the distant one.

Outline

(I) Review of local hidden-variable models

- Outcome Independence and Parameter Independence
- Action vs. passion at a distance

(II) Analyzing passion at a distance

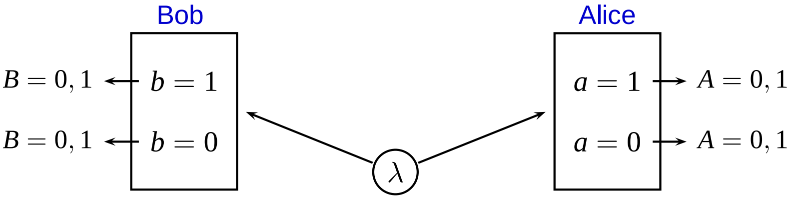
- Rewriting the CHSH inequality
- Introducing the Guessed Information (GI)
- Introducing the Transmitted Information (TI)
- On what it takes to violate the CHSH inequality

(III) Comparing action vs. passion at a distance

- On what it takes to violate the CHSH inequality revisited

(IV) Discussion and conclusion

Section I: Local realism and hidden variables



1. One assumes that the particle pair and other relevant degrees of freedom are captured in some physical state $\lambda \in \Lambda$ ('beables').

2. The model gives the probability for obtaining outcomes A, B when measuring a, b on a system in the state λ: $\quad P(A, B|a, b, \lambda)$.

3. Empirically accessible probabilities of outcomes are obtained by averaging over some probability density on λ:

$$P(A, B|a, b) = \int_\Lambda P(A, B|a, b, \lambda)\rho(\lambda|a, b)d\lambda.$$

Conditions imposed on the model

1. Factorisability (Bell called this ‘local causality’):

$$P(A, B|a, b, \lambda) = P(A|a, \lambda)P(B|b, \lambda).$$

2. Independence of the Source (IS): $\rho(\lambda|a, b) = \rho(\lambda)$.

   Also called: ‘freedom of choice‘ ; the assumption of ’free variables’.

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Discussion: This ’freedom’ assumption justifies the distinction between outcomes and settings (although this distinction itself is not fundamental): $\Longrightarrow$ The outcomes are always dependent variables, whereas the settings should be independent variables.

▶ This is the cause of the asymmetry between outcomes and settings in a Bell-type analysis. The present work also further investigates this asymmetry.

Bell's theorem

Consequences of the assumptions:

Factorisability $\wedge$ 'freedom' $\Longrightarrow P(A,B|a,b) = \int_\Lambda P(A|a,\lambda)P(B|b,\lambda)\rho(\lambda)d\lambda$

Thus all correlations are local correlations.

$\Longrightarrow$ CHSH inequality is obeyed:

$$|\langle a_0 b_0\rangle + \langle a_0 b_1\rangle + \langle a_1 b_0\rangle - \langle a_1 b_1\rangle| \leq 2$$

Carving up factorisability

Jarrett / Shimony introduced finer distinctions that together imply factorisability.

- **Parameter Independence (PI)**:

$$P(A|a,b,\lambda) = P(A|a,\lambda) \quad \text{and} \quad P(B|a,b,\lambda) = P(B|b,\lambda).$$

- **Outcome Independence (OI)**:

$$P(A|a,b,B,\lambda) = P(A|a,b,\lambda) \quad \text{and} \quad P(B|a,b,A,\lambda) = P(B|a,b,\lambda).$$

PI $\wedge$ OI $\implies$ Factorisability: $P(A,B|a,b,\lambda) = P(A|a,\lambda)P(B|b,\lambda)$.

Experimental metaphysics

Assuming ‘freedom of choice’ it must be that either OI or PI is not obeyed in violations of the CHSH inequality

In the doctrine of **Experimental Metaphysics** it is mostly violation of the latter condition (i.e. OI) that is supposed to be responsible for the violation of the CHSH inequality.

¬ OI: $P(A|a,b,B,\lambda) \neq P(A|a,b,\lambda)$

It is then said: Bob, knowing his outcome, can predict Alice’s outcome better than was possible just based on the state λ and the settings. But he cannot warn Alice because the outcome is not under his control.

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$\neg$ OI: $P(A|a,b,B,\lambda) \neq P(A|a,b,\lambda)$

It is then said: Bob, knowing his outcome, can predict Alice’s outcome better than was possible just based on the state λ and the settings. But he cannot warn Alice because the outcome is not under his control.

- It is argued that this is not an instance of action at a distance but of some innocent ’passion at a distance’, i.e., an increase in non-local predictability: one passively comes to know more about the faraway situation, but one cannot actively change it.

But this is not passion at a distance at all

$\neg$ PI: $$P(A|a,b,\lambda) \neq P(A|a,\lambda)$$

$\neg$ OI: $$P(A|a,b,B,\lambda) \neq P(A|a,b,\lambda)$$

Violations of PI and OI show a dependence of a *local* probability on a *non-local* outcome or setting.

- Upon closer scrutiny, both PI and OI do **not** address the possibility of ‘coming to know’ the *non-local* outcomes or settings.

More technically: the conditions are not about an increase in non-local predictability because of the availability of non-local information.

Therefore, they do not deal with passion at a distance at all, and in fact, there has not been a satisfactory analysis of it anywhere. Such an analysis will be given here.

Section II: Analyzing passion at a distance

[Joint work with Pawlowski et al. (arXiv:0903.5042)]

Question: what kind of information—about the distant measurement setting or the outcome or both—and which amount of it has to be non-locally available to simulate the violation of the CHSH inequality within the framework of non-local realistic models.

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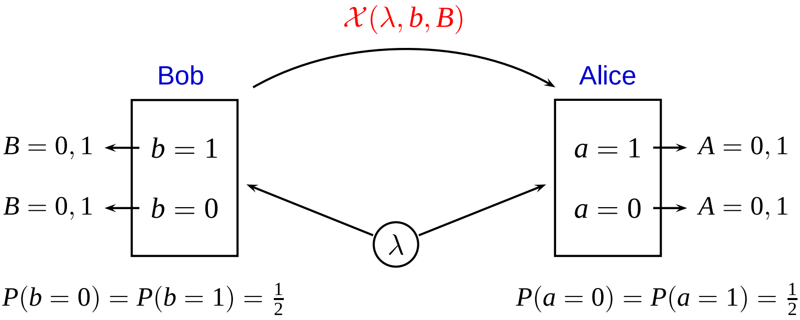
- Here it is assumed that the information becomes available through one-way classical communication.

Although, the results do not depend on there being an actual communication process.

$\Longrightarrow$ Alternatively, think of the communicated 'message' as extra information that tells about Bob's situation and which is just *somehow* available to Alice.

one-way communication paradigm

Consider the standard Bell-setup, but augmented with one-way classical communication:



Note: 'freedom of choice' will be assumed: Alice and Bob receive hidden variables λ which are independent of the choice of the settings.

one-way communication paradigm

1. Bob generates the *message* $\mathcal{X}$ which depends on λ, b and B.
2. It is assumed that the exact mechanism how B and $\mathcal{X}$ are generated by Bob is known to Alice.
3. Alice uses her optimal strategy, based on the knowledge of her setting a, the shared hidden variables λ, and the message $\mathcal{X}$, to produce her outcome A in order to maximally violate the CHSH inequality.

Rewriting the CHSH inequality

The CHSH inequality

$$\langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle \leq 2$$

can be written in terms of joint probabilities using the following relations

$$P(A, B|a, b) = \frac{1}{4}[1 + (-1)^{A \oplus B}\langle ab \rangle + (-1)^{A}\langle a \rangle + (-1)^{B}\langle b \rangle]$$

It then becomes :

$$\sum_{a,b=0}^{1} P(A \oplus B = ab|a, b) \leq 3, \qquad (\oplus \bmod 2)$$

Let us now define:

$$P(A = B|a = 0) := \sum_{b'=0}^{1} P(b')\, P(A = B|a = 0, b')$$

$$P(A = B \oplus b|a = 1) := \sum_{b'=0}^{1} P(b')P(A = B \oplus b|a = 1, b')$$

and remembering $P(b = 0) = P(b = 1) = \frac{1}{2}$, gives the identity

$$P(A \oplus B = ab|a, b) = 2\, P(A = B|a = 0) + 2\, P(A = B \oplus b|a = 1).$$

- Thus the CHSH ineq. becomes:

$$\boxed{\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4}}$$

called the ‘CHSH inequality from Alice’s perspective’.

On the 'CHSH inequality from Alice's perspective'

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) \leq \frac{3}{4},$$

These probabilities can be interpreted as a measure of information Alice has about Bob's settings and outcomes.

- To do so, the Guessed Information Π is introduced:

$$\Pi(\mathcal{X} \to \mathcal{Y}) = \sum_i P(\mathcal{X} = i) \max_j \left[P(\mathcal{Y} = j|\mathcal{X} = i) \right]$$

where $\mathcal{X}$ takes values $i = 1, ..., X$ and $\mathcal{Y}$ values $j = 1, ..., Y$.

On the Guessed Information

1. The value of $\Pi(\mathcal{X} \to \mathcal{Y})$ gives the average probability to correctly guess $\mathcal{Y}$ knowing the value of $\mathcal{X}$.
2. Its maximal value is 1 and corresponds to the situation in which $\mathcal{Y}$ is fully specified by $\mathcal{X}$.
3. The minimal value of $\Pi(\mathcal{X} \to \mathcal{Y})$ equals $\frac{1}{Y}$ and corresponds to the situation in which $\mathcal{X}$ reveals no information about $\mathcal{Y}$.
4. GI reaches its minimum when the mutual information is $I(\mathcal{X} : \mathcal{Y}) = 0$, and it is maximal when $I(\mathcal{X} : \mathcal{Y}) = \log Y$.

Example:

(i) 'freedom of choice': necessarily $\Pi(\lambda \to b) = \frac{1}{2}$,

(ii) but note that $\Pi(\lambda \to B) > \frac{1}{2}$ is possible.

- ▶ The source of the asymmetry between settings and outcomes.

Violating the CHSH inequality

$$\frac{1}{2}P(A=B|a=0)+\frac{1}{2}P(A=B\oplus b|a=1)\leq\frac{3}{4},$$

1) Alice tries to violate this CHSH inequality using the received message $\mathcal{X}$.

2) To do so she must maximize the probabilities in this inequality.

3) These not only involve the local information A and a, but in general also some function $f(B,b)$ of the outcome B and setting b at Bob's side.

4) Any result she would obtain can never be larger than the average probability for Alice to correctly guess $f(B,b)$ given λ her own mechanism, and the message $\mathcal{X}$

- The probability is upperbounded by $\Pi(\lambda,\mathcal{X}\to f(B,b))$.

Bounding the probabilities

The probability that, for example, the outcomes are equal ($A = B$), when Alice's setting happens to be $a = 0$, is thus at most as large as the guessed information $\Pi(\lambda, \mathcal{X} \to B)$:

$$P(A = B|a = 0) \leq \Pi(\lambda, \mathcal{X} \to B).$$

Similarly,

$$P(A = B \oplus b|a = 1) \leq \Pi(\lambda, \mathcal{X} \to B \oplus b).$$

That is, the probability that, for example, the outcomes are $A = B \oplus b$, when Alice's setting is $a = 1$, is at most as large as the guessed information $\Pi(\lambda, \mathcal{X} \to B \oplus b)$.

Note: In optimal cases there will be equality.

On what it takes to violate the CHSH inequality

In order for a violation of the CHSH inequality to occur it must of course be that

$$\frac{1}{2}P(A = B|a = 0) + \frac{1}{2}P(A = B \oplus b|a = 1) > \frac{3}{4},$$

which, given the previous results, implies the following *necessary condition* for a violation of the CHSH inequality:

$$\frac{1}{2}\Pi(\lambda, \mathcal{X} \to B) + \frac{1}{2}\Pi(\lambda, \mathcal{X} \to B \oplus b) > \frac{3}{4}$$

Finally, we are in the position to assess ‘passion at a distance’.

Assessing ‘passion at a distance’

$$\text{Distant Setting Ignorance (DSI):} \quad \Pi(\lambda, \mathcal{X} \to b) = \frac{1}{2}$$

$$\text{Distant Outcome Ignorance (DOI):} \quad \Pi(\lambda, \mathcal{X} \to B) = \frac{1}{2}$$

These deal with what can be non-locally predicted. In contrast to OI and PI, they are not about non-locally influencing something.

- The appropriate conditions for assessing ‘passion at a distance’.

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- The appropriate conditions for assessing 'passion at a distance'.

It will be shown that a *necessary* condition for violation is that *both* information about the setting and about the outcome produced at one lab *must be available* at the distant lab.

That is, both of the above conditions must be violated.

Both information about the setting and outcome must be available

Necessary for violation: $\frac{1}{2}\Pi(\lambda, \mathcal{X} \to B) + \frac{1}{2}\Pi(\lambda, \mathcal{X} \to B \oplus b) > \frac{3}{4}$

Both information about the setting and outcome must be available.

1. If no outcome information is available, i.e. $\Pi(\lambda, \mathcal{X} \to B) = \frac{1}{2}$, the left-hand side cannot exceed $\frac{3}{4}$.

   $\implies \quad \Pi(\lambda, \mathcal{X} \to B) > \frac{1}{2}$

2. Analogously it must be that $\Pi(\lambda, \mathcal{X} \to B \oplus b) > \frac{1}{2}$.

   To prove that setting information is also necessary, note that if one knows both B and $B \oplus b$, one also knows b.

   This can be made formal: $\Pi(\lambda, \mathcal{X} \to b) > \frac{1}{2}$.

What information must be available, over and above λ?

One may further ask if

1. the available information comes from the source via the shared hidden variable λ (which acts as a common cause),
2. or should it be transmitted through the message $\mathcal{X}$?

▶ This calls for a further analysis of what information has to be transmitted via the message $\mathcal{X}$, over and above the information in the hidden variable λ.

Information Transmission

Consider the so-called 'Transmitted Information' (TI), which is the difference of the averaged probability of correctly guessing the value of the variable $\mathcal{Y}$ when knowing $\mathcal{X}$ and λ, and the one when knowing only λ:

$$\Delta_\lambda(\mathcal{X} \to \mathcal{Y}) = \Pi(\lambda, \mathcal{X} \to \mathcal{Y}) - \Pi(\lambda \to \mathcal{Y}), \quad \in [0, 1 - \frac{1}{Y}].$$

Its lowest value indicates: transmission of $\mathcal{X}$ does not increase Alice's ability to guess the correct value of $\mathcal{Y}$.

$\implies$ $\mathcal{X}$ carries no new information about $\mathcal{Y}$ (that is not already available to Alice through λ).

Information Transmission

λ-exceeding distant setting information transmission (DSIT$_{\neg\lambda}$):

$$\Delta_\lambda(\mathcal{X} \rightarrow b) > 0$$

λ-exceeding distant outcome information transmission (DOIT$_{\neg\lambda}$):

$$\Delta_\lambda(\mathcal{X} \rightarrow B) > 0$$

- The ’passion at a distance’ not already accounted for by λ.

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- The 'passion at a distance' not already accounted for by λ.

In violations of the CHSH inequality:

1. It is possible that the information about the **outcome** can be obtained solely from the shared hidden variables:

   It can be that $\Delta_\lambda(\mathcal{X} \to B) = 0$.

2. However, given 'freedom', the information about the **setting** must be communicated, implicit or explicit, non-locally:

   It must be that $\Delta_\lambda(\mathcal{X} \to b) > 0$.

‘Freedom of choise’ and the asymmetry

‘Freedom of choice’, i.e $\Pi(\lambda \to b) = \frac{1}{2}$, forces this asymmetry between the outcome and setting information.

(A) It leads to

$$\Delta_\lambda(\mathcal{X} \to b) = \Pi(\lambda, \mathcal{X} \to b) - \frac{1}{2}.$$

1. We see that $\Delta_\lambda(\mathcal{X} \to b) = 0$ leads to $\Pi(\lambda, \mathcal{X} \to b) = \frac{1}{2}$.
2. But we have seen that this implies no violation of the CHSH inequality.
3. Thus the Setting Information Transmission must be greater than zero: $\Delta_\lambda(\mathcal{X} \to b) > 0$.

‘Freedom of choise’ and the asymmetry

(B) On the other hand, there is no assumption corresponding to freedom-of-choice regarding the outcomes.

- no reason to demand $\Pi(\lambda \to B) = \frac{1}{2}$.

Instead, one has

$$\Pi(\lambda, \mathcal{X} \to B) = \Delta_\lambda(\mathcal{X} \to B) + \Pi(\lambda \to B).$$

Since $\Pi(\lambda, \mathcal{X} \to B) > \frac{1}{2}$, it must be that either $\Delta_\lambda(\mathcal{X} \to B) = 0$ or $\Pi(\lambda \to B) = \frac{1}{2}$, but not both.

Results: always $\Delta_\lambda(\mathcal{X} \to b) \geq 0$ and either $\Delta_\lambda(\mathcal{X} \to B) = 0$ or $\Pi(\lambda \to B) = \frac{1}{2}$, but not both.

What it takes to violate the CHSH inequality

Condition holds	violation of CHSH possible?
$\Pi(\lambda, \mathcal{X} \to b) = \frac{1}{2}$	No
$\Pi(\lambda, \mathcal{X} \to B) = \frac{1}{2}$	No
$\Pi(\lambda \to b) = \frac{1}{2}$	Yes ('freedom')
$\Pi(\lambda \to B) = \frac{1}{2}$	Yes*
$\Delta_\lambda(\mathcal{X} \to b) = 0$	No
$\Delta_\lambda(\mathcal{X} \to B) = 0$	Yes*

*: for violation either $\Pi(\lambda \to B) = \frac{1}{2}$ or $\Delta_\lambda(\mathcal{X} \to B) = 0$ can hold, but not both.

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$\Pi(\lambda \rightarrow b) = \frac{1}{2}$	Yes ('freedom')
$\Pi(\lambda \rightarrow B) = \frac{1}{2}$	Yes*
$\Delta_\lambda(\mathcal{X} \rightarrow b) = 0$	No
$\Delta_\lambda(\mathcal{X} \rightarrow B) = 0$	Yes*
OI	Yes**
PI	Yes**

Examples

– Toner and Bacon (2003): They simulate the quantum singlet state by communicating 1 classical bit. But if only maximal violation of CHSH is to be simulated, then $\mathcal{X}$ contains only 0.736 bits. (arxiv:0903.5042)

– Leggett-style model of Gröblacher et al. (2007): a unit vector is being send.

– Bohmian mechanics (1952): a subtle issue. There is no message sent. But the setting information is non-locally present through the wavefunction which acts as a guiding field.

▶ In all cases it is setting information which is nonlocally available.

While either OI or PI can be obeyed in models giving a violation of the CHSH inequality, one needs

- **both** $\Pi(\mathcal{X}, \lambda \to b) > \frac{1}{2}$ and $\Pi(\mathcal{X}, \lambda \to B) > \frac{1}{2}$,
- **always** $\Delta_\lambda(\mathcal{X} \to b) > 0$,
- and **either** $\Pi(\lambda \to B) = \frac{1}{2}$ **or** $\Delta_\lambda(\mathcal{X} \to B) = 0$, but not both.

This shows the conceptual difference between action and passion at a distance, i.e. between the conditions that focus on

1. non-local (in)dependence (OI, PI).
2. non-local increase of predictability (Guessed information), possibly over and above what the hidden variables allow one to predict (Transmitted Information).

Section IV: Conclusion and discussion

1) All this can be taken out of the one-way communication paradigm. Instead of ’transmission of a message’ think of ’ extra information being available to Alice’. (e.g. Bohmian mechanics)

2) I believe this allows for progress in the field of Experimental Metaphysics.

3) As a side effect, it can be noted that these results are also relevant for quantifying the classical resources needed to simulate quantum communication and computation protocols.

4) This analysis tried to trace the asymmetry between outcomes and settings so as to originate from the ‘freedom of choice’ assumption.