Parts and Wholes

On some results from my dissertation

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Prospects & Introduction

I will reconsider the well-known (local) hidden variable program.

Some **elementary** investigations and new results are presented that I believe to have general repercussions. These are intended to deepen our understanding of what it takes to violate local realism and/or the CHSH inequality and how this relates to signaling vs. no-signaling correlations.

Methodological morale:

> *Now it is precisely in cleaning up intuitive ideas for mathematics that one is likely to throw out the baby with the bathwater.*
>
> *J.S. Bell; ‘La nouvelle cuisine’, 1990.*

Outline

(I) Review of (local) hidden-variable models

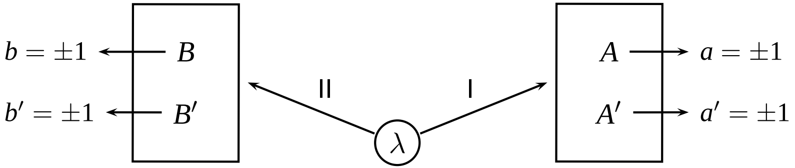
(II) Some results
CHSH inequality revisited
Surface vs. subsurface level
Discerning no-signaling correlations

(III) Deep hidden variables and (in)completeness
– Consequences of the existence of a deeper level

(IV) Conclusion and outlook

Local realism and hidden variables

Setup of the *Gedankenexperiment*:



- Locality: the idea that there exists no spacelike causation.
- Realism: the idea that (i) physical systems exist independently and (ii) posses intrinsic properties describable by states.
- Free variables: the settings used to measure observables can be chosen freely, i.e., this excludes conspiracy theories (e.g., super-determinism) as well as retro-causal interactions.

1. One assumes that the particle pair and other relevant degrees of freedom are captured in some physical state $\lambda \in \Lambda$ ('beables').

2. Further requirement that is often used: λ provides a complete (or *full* or *total* or *exhaustive*) specification of the state of the situation in question.

   $\longrightarrow$ In need of clarification ... to be clarified later.

3. The hidden variable model gives the probability for obtaining outcomes a, b when measuring A, B on a system in the state λ:

$$P(a, b|A, B, \lambda).$$

4. Empirically accessible probabilities of outcomes are obtained by averaging over some probability density on λ:

$$P(a, b|A, B) = \int_\Lambda P(a, b|A, B, \lambda)\rho(\lambda|A, B)d\lambda.$$

Conditions

– Factorisability (Bell called this 'local causality'):

$$P(a,b|A,B,\lambda) = P(a|A,\lambda)P(b|B,\lambda).$$

– Independence of the Source (IS): $\rho(\lambda|A,B) = \rho(\lambda)$.

Consequences of the assumptions:

$$\text{Factorisability} \wedge \text{IS} \implies P(a,b|A,B) = \int_\Lambda P(a|A,\lambda)P(b|B,\lambda)\rho(\lambda)d\lambda$$

$$\implies \text{CHSH inequality is obeyed.}$$

$$|\langle AB\rangle_{\text{lhv}} + \langle AB'\rangle_{\text{lhv}} + \langle A'B\rangle_{\text{lhv}} - \langle A'B'\rangle_{\text{lhv}}| \leq 2$$

Carving up factorisability

Jarrett / Shimony introduce finer distinctions that together imply factorisability.

– Parameter Independence (PI):

$$P(a|A, B, \lambda) = P(a|A, \lambda) \text{ and } P(b|A, B, \lambda) = P(b|B, \lambda).$$

– Outcome Independence (OI):

$$P(a, b|A, B, \lambda) = P(a|A, B, \lambda)P(b|A, B, \lambda).$$

PI $\wedge$ OI $\implies$ Factorisability: $P(a, b|A, B, \lambda) = P(a|A, \lambda)P(b|B, \lambda)$.

PART 2: Further definitions

Surface probabilities: $P(a, b|A, B)$

Determined via measurement of relative frequencies.

Subsurface probabilities: $P(a, b|A, B, \lambda)$

Generally inaccessible, conditioned on hidden variables.

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- Definitions of different kinds of bi-partite surface correlations:

a) **Local**: $P(a,b|A,B) = \int_\Lambda d\lambda\, \rho(\lambda)\, P(a|A,\lambda) P(b|B,\lambda)$.

b) **No-signaling**: $P(a|A)^B = P(a|A)^{B'} := P(a|A)$

where $P(a|A)^B = \sum_b P(a,b|A,B)$, etc.

c) **Quantum**: $P(a,b|A,B) =$Tr$[\,M_a^A \otimes M_b^B\, \rho\,]$, $\sum_a M_a^A = \mathbb{1}$.

On the CHSH inequality in Quantum Mechanics

Consider the **CHSH polynomials**: $\mathcal{B}$ and $\mathcal{B}'$, where

$$\mathcal{B} = AB + AB' + A'B - A'B' \qquad (1)$$

- Then, all quantum states must obey:

$$\max_{A,A',B,B'} \langle \mathcal{B} \rangle_\rho^2 + \langle \mathcal{B}' \rangle_\rho^2 \leq 8, \quad \forall \rho \in \mathcal{D}, \qquad (2)$$

This implies the Tsirelson inequality:

$$\max_{A,A',B,B'} |\langle \mathcal{B} \rangle_\rho|, |\langle \mathcal{B}' \rangle_\rho| \leq 2\sqrt{2}, \quad \forall \rho \in \mathcal{D}. \qquad (3)$$

Separable states must obey the more stringent bound:

$$\max_{A,A',B,B'} |\langle \mathcal{B} \rangle_\rho|, |\langle \mathcal{B}' \rangle_\rho| \leq 2, \quad \forall \rho \in \mathcal{D}_{\text{sep}}. \qquad (4)$$

Orthogonal Measurements

The maximal quantum bounds are only obtainable using entangled states and when choosing **Orthogonal measurements** (= anti-commuting; $\{A, A'\} = 0$, $\{B, B'\} = 0$).

Now, for any spin-$\frac{1}{2}$ state ρ on $\mathcal{H} = \mathbb{C}^2$, and any orthogonal triple of spin components A, A' and A'', one has

$$\langle A \rangle_\rho^2 + \langle A' \rangle_\rho^2 + \langle A'' \rangle_\rho^2 \leq 1. \tag{5}$$

$\Longrightarrow$ But then separable states must obey a sharper quadratic inequality:

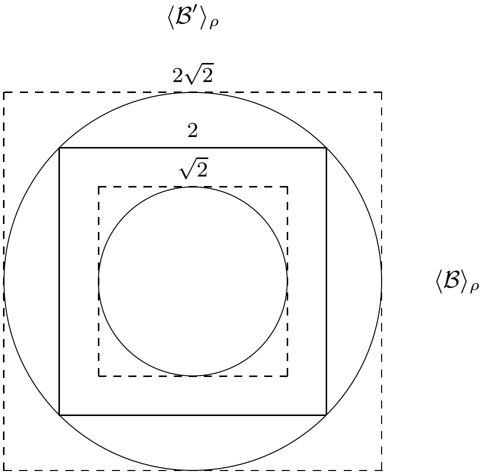
$$\max_{A \perp A', B \perp B'} \langle \mathcal{B} \rangle_\rho^2 + \langle \mathcal{B}' \rangle_\rho^2 \leq 2, \quad \forall \rho \in \mathcal{D}_{\text{sep}}, \tag{6}$$

which in turn gives the linear inequalities:

$$\max_{A \perp A', B \perp B'} |\langle \mathcal{B} \rangle_\rho|, |\langle \mathcal{B}' \rangle_\rho| \leq \sqrt{2}, \quad \forall \rho \in \mathcal{D}_{\text{sep}}. \tag{7}$$

- A factor $\sqrt{2}$ stronger than the original CHSH bound of 2.

On the CHSH inequality



Comparing to LHV theories

It is interesting to ask whether one can obtain a similar stronger inequality as $|\langle\mathcal{B}\rangle_\rho| \leq \sqrt{2}$ in the context of local hidden-variable theories, for which we know $|\langle\mathcal{B}\rangle_{\text{lhv}}| \leq 2$ (CHSH, 1969) .

The assumption to be added to such an LHV theory is the requirement that for any orthogonal choice of A, A' and A'' and for every given λ we have the analog of (5) which is

$$\langle A\rangle^2_{\text{lhv}} + \langle A'\rangle^2_{\text{lhv}} + \langle A''\rangle^2_{\text{lhv}} = 1, \tag{8}$$

where $\langle A\rangle_{\text{lhv}} = \sum_{a=\pm 1} a\, P(a|A,\lambda)$, etc.

- But a requirement like (8) is by no means obvious for a local hidden-variable theory.

Indeed, as has often been pointed out, such a theory may employ a mathematical framework which is completely different from quantum theory. There is no *a priori* reason why the orthogonality of spin directions should have any particular significance in the hidden-variable theory, and why such a theory should confirm to quantum mechanics in reproducing $\langle A\rangle^2_{\rm lhv} + \langle A'\rangle^2_{\rm lhv} + \langle A''\rangle^2_{\rm lhv} = 1$ if one conditionalizes on a given hidden-variable state.

(One is reminded here of Bell's critique on von Neumann's 'no-go theorem'.)

Thus, the additional requirement would appear entirely unmotivated within an LHV theory.

It thus appears that testing for entanglement within quantum theory and testing quantum mechanics against the class of all LHV theories are not equivalent issues.

Of course, this conclusion is not new: Werner already constructed an explicit LHV model for a specific two-qubit entangled state. Consider the so-called Werner states: $\rho_W = \frac{1-p}{4}\mathbb{1} + p|\psi^-\rangle\langle\psi^-|$, $p \in [0,1]$. Werner has shown that these states are entangled if $p > 1/3$, but nevertheless possess an LHV model for $p = 1/2$.

The above inequality suggests that the phenomenon exhibited by this Werner state is much more ubiquitous, i.e., that many more entangled two-qubit states have an LHV model. This has been shown to be indeed the case.

Surface vs. Subsurface Levels

Subsurface:

- $\text{OI} \wedge \text{PI} \implies$ Factorisability
- Determinism $\implies$ OI

(i) Deterministic hidden variables and violation of Factorisability implies violation of PI. (e.g. Bohmian mechanics)

(ii) PI and violation of Factorisability implies indeterminism at the hidden-variable level.

Surface analogs of (i) and (ii):

(iii) Any non-local correlation that is deterministic must be signaling.

(iv) Any non-local correlation that is no-signaling must be indeterministic, i.e., the outcomes are only probabilistically predicted. (e.g., Bohm)

Proof: Any deterministic no-signaling correlation must be local. [cf. Masanes et al. (2006)]

- Consider a deterministic probability distribution $P_{\text{det}}(a, b|AB)$.

$\Longrightarrow$ The outcomes a and b are deterministic functions of A and B: $a = a[A, B]$ and $b = b[A, B]$.

- Suppose it is a no-signaling distribution, then

$$\begin{aligned} P_{\text{det}}(a, b|AB) &= \delta_{(a,b),(a[(A,B],b[A,B])} = \delta_{a,a[A,B]}\delta_{b,b[A,B]} \\ &= P(a|A, B)P(b|A, B) = P(a|A)P(b|B). \end{aligned}$$

This is a local distribution and therefore any deterministic no-signaling correlation must be local.

Determinism, yet indeterminism

Now again consider Bohmian mechanics: because it obeys no-signaling and gives rise to non-local correlations (since it violates the CHSH inequality) it must predict the outcomes only probabilistically.

In other words, although fundamentally deterministic it must necessarily be predictively indeterministic.

▶ Thus no Bohmian demon can have perfect control over the hidden variables and still be non-local and no-signaling at the surface (as QM requires).

• This is not specific to Bohmian mechanics: any deterministic theory that obeys no-signaling and gives non-local correlations must have the same feature: it must predict the outcomes of measurement indeterministically.

Discerning no-signaling correlations

We have seen that requiring no-signaling in conjunction with some other constraint has strong consequences.

- But what if we solely require no-signaling? Can we find a non-trivial constraint that follows from no-signaling alone?

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The CHSH inequality does not suffice to discern no-signaling correlations because they can maximally violate it up to a value of 4 (e.g., PR-boxes). *But an analogue does*:

$$|\langle AB\rangle_{\mathrm{ns}} + \langle A'B\rangle_{\mathrm{ns}} + \langle A\rangle^{B}_{\mathrm{ns}} - \langle A'\rangle^{B}_{\mathrm{ns}}| \leq 2.$$

Here $\langle A\rangle^{B}_{\mathrm{ns}} := \sum_{a,b} a \int_{\Lambda} d\lambda \rho(\lambda|A,B) P(a,b|A,B,\lambda)$, and $P(a|A)^{B} = P(a|A)^{B'} := P(a|A)$.

Reproducing perfect singlet-state correlations

$$\forall\,\vec{a},\,\vec{b}:\quad \langle\vec{a}\,\vec{b}\rangle = -1,\quad \text{when}\quad \vec{a} = \vec{b}$$
$$\forall\,\vec{a},\,\vec{b}:\quad \langle\vec{a}\,\vec{b}\rangle = 1,\quad \text{when}\quad \vec{a} = -\vec{b}$$

- The no-signaling inequalities give two non-trivial constraints:

$$\langle\vec{a}\rangle^{I}_{\text{ns}} + \langle\vec{a}\rangle^{II}_{\text{ns}} = 0$$
$$\langle-\vec{a}\rangle^{I}_{\text{ns}} = -\langle\vec{a}\rangle^{I}_{\text{ns}}$$

This states that the marginal expectation values for party I and II must add up to zero for measurements in the same direction, and individually they must be odd functions of the settings.

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This states that the marginal expectation values for party I and II must add up to zero for measurements in the same direction, and individually they must be odd functions of the settings.

- Consequently, any model reproducing the singlet state perfect (anti-) correlations and which does not obey either one (or both) of these conditions must be signaling.

  - ▶ In case both systems are treated the same, i.e., $\langle \vec{a} \rangle^{I}_{\text{ns}} = \langle \vec{a} \rangle^{II}_{\text{ns}}$, the marginal expectation values must vanish: $\langle \vec{a} \rangle^{I}_{\text{ns}} = \langle \vec{a} \rangle^{II}_{\text{ns}} = 0$.

PART 3: Deep hidden variables

- Back to the LHV program.

– Independence of the Source (IS): $\rho(\lambda|A,B) = \rho(\lambda)$.

– Parameter Independence (PI):

$$P(a|A,B,\lambda) = P(a|A,\lambda) \text{ and } P(b|A,B,\lambda) = P(b|B,\lambda).$$

– Outcome Independence (OI):

$$P(a,b|A,B,\lambda) = P(a|A,B,\lambda)P(b|A,B,\lambda).$$

Motivations for the conditions:

IS: via the notion of free variables.

PI: via invoking locality.

OI: opinions differ. Some use locality, others rely on realism or invoke some other idea.

Motivating OI: $P(a, b|A, B, \lambda) = P(a|A, B, \lambda)P(b|A, B, \lambda)$

I take OI to follow from a **completeness** or **sufficiency** condition that encodes that our theory takes into account all there is to know, i.e., no relevant degrees of freedom are left out.

- λ (together with the settings A and B) is complete, i.e., sufficient for the (probability of obtaining) outcomes a and b. To be further clarified later.

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– Shimony uses an appeal to locality.

– Elby, Brown and Foster claim that 'Jarrett completeness [OI] follows from natural assumptions about locality and causality'.

– Others use Reichenbach's principle of common cause or Helmann's Stochastic Einstein Locality (cf. Butterfield).

– Rejecting of Jarrett's project. Norsen: 'The whole motivation of Jarrett's project . . . is based on a fundamental confusion'. cf. Westman.

Completeness and deeper hidden variables

- How to formalize an idea of completeness?

Consider a theory that posits that apart from the hidden variable λ there is also **a deeper level hidden variable** ξ and that all the probabilities $P(a,b|A,B,\lambda)$ are actually averages over the additional variable:

$$P(a,b|A,B,\lambda) = \int P(a,b|A,B,\lambda,\xi)\,\rho(\xi|\lambda)\,d\xi.$$

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How deep can one go? Any stochastic hidden-variable model can be supplemented adding additional variables.
Mathematically one can go as deep as to get a fully deterministic theory: all $P(a, b|A, B, \lambda, \xi)$ are either 0 or 1.

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How deep can one go? Any stochastic hidden-variable model can be supplemented adding additional variables. Mathematically one can go as deep as to get a fully deterministic theory: all $P(a, b|A, B, \lambda, \xi)$ are either 0 or 1.

- However, it is important to realize that such a procedure only makes sense if one physically assumes that the stochastic model is *incomplete* since a relevant deeper hidden-variable description is assumed to exist.

Higher vs. deeper level

Suppose such a deeper level exists. If conditions hold at one level, they need not also hold at another level!

Examples:

(A)

- Orthodox QM: $\lambda = |\,\psi\rangle \implies$ PI holds, OI fails

- Bohmian mechanics: $\lambda = (|\,\psi\rangle, \vec{x}_1, \vec{x}_2) \implies$ OI holds, PI fails.
  (deeper level hidden variables, deterministic.)

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(B) Leggett's 2003 model:

- Deepest deterministic level: $\lambda = (\gamma, \vec{u}, \vec{v}) \implies$ PI fails, OI holds

- On the level of $(\vec{u}, \vec{v})$; average over $\gamma \implies$ OI fails, PI holds.

(Note: in Leggett's model γ does no work at all. All his physical assumptions are at the $(\vec{u}, \vec{v})$ level.)

This shows explicitly that parameter dependence (violation of PI) at the deeper deterministic hidden-variable level does not show up as parameter dependence at the higher hidden-variable level, but as outcome dependence, i.e., as a violation of OI.

In other words, violation of OI could be a sign of a violation of (deterministic) PI at a deeper hidden-variable level.

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- In the deterministic case the feature above is generic: a violation of OI implies a violation of deterministic PI at the deeper hidden-variable level where the model is deterministic.

General picture

Higher level		Deeper level
PI holds	$\not\Rightarrow$ $\Leftarrow$	PI holds
OI holds	$\not\Rightarrow$ $\not\Leftarrow$	OI holds

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– <u>From higher level to deeper level</u>:

OI and PI at higher level need not imply OI and PI at deeper level, respectively. And the converse: violations of OI and of PI at the deeper level need not show up at higher level.

- But this is to be expected: Averaging over some variables may wash out correlations and dependencies.

Higher level		Deeper level
PI holds	$\not\Longrightarrow$ $\Longleftarrow$	PI holds
OI holds	$\not\Longrightarrow$ $\not\Longleftarrow$	OI holds

– From deeper level to higher level:

PI at deeper level implies PI at higher level.

- Independence at a deeper level is conserved by averaging. Averaging cannot create any dependencies.

OI at deeper level does not imply OI at higher level.

- If regarded as a completeness condition this is to be expected since one generally leaves out some relevant hidden variables. Completeness then is given up.

Cause of the assymetry

Mathematically:

- Due to the non-convexity of factorisation conditions such as OI. They do no longer hold under convex combinations and decompositions.
- Whereas: independence conditions such as PI remain to hold under convex combinations (but of course not under convex decompositions).

Recall:

OI: $P(a, b|A, B, \lambda) = P(a|A, B, \lambda)P(b|A, B, \lambda)$

PI: $P(a|A, B, \lambda) = P(a|A, \lambda)$ and $P(b|A, B, \lambda) = P(b|B, \lambda)$

Being explicit about completeness

We thus see that which conditions are obeyed and which are not depends on the level of consideration.

- A conclusive picture therefore depends on which hidden-variable level is considered to be fundamental.

But usually this is not mentioned. This is unfortunate, it hinders interpretation.

Many use such words as *complete, full, exhaustive, sufficient.* But none give a specific definition, and moreover any such a model can be considered to give a complete specification, namely of all variables that happen to feature in the model.

- But this misses the point. The point is whether specifying extra hidden variables (that perhaps are not yet in our theory) is in fact redundant.

$\Longrightarrow$ One should address whether specifying extra hidden variables is in fact redundant. I therefore propose the condition:

Completeness (COMP): A hidden variable theory is complete if for all a, b and *all possible* extra hidden variables ξ (other than the settings ('free variables') A, B and hidden variable λ) the following holds:

$$(a)\ P(a,b|A,B,\lambda,\xi) = P(a,b|A,B,\lambda), \tag{9}$$

$$(b)\ P(a|A,\lambda,\xi) = P(a|A,\lambda) \ \text{ and } \ P(b|B,\lambda,\xi) = P(b|B,\lambda). \tag{10}$$

This formalises the notion of 'completeness of a hidden-variable model' in a probabilistic framework.

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From COMP it follows for the marginals that

$$P(a|A, B, \lambda, \xi) = P(a|A, B, \lambda), \tag{11}$$

$$P(b|A, B, \lambda, \xi) = P(b|A, B, \lambda). \tag{12}$$

Being explicit about completeness

- If COMP holds: introducing a deeper level has no effect.

Higher level		Deeper level
OI holds	$\Longleftrightarrow$	OI holds
PI holds	$\Longleftrightarrow$	PI holds

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Proof:

• OI: deep $\Longrightarrow$ high. Assume COMP(1) and OI at the deep level:

$$P(a,b|A,B,\lambda) \overset{\text{COMP}(1)}{=} P(a,b|A,B,\lambda,\xi) \overset{\text{OI}_{\text{deep}}}{=}$$
$$P(a|A,B,\lambda,\xi)P(b|A,B,\lambda,\xi) \overset{\text{COMP}(1)}{=} P(a|A,B,\lambda)P(b|A,B,\lambda).$$

• PI: high $\Longrightarrow$ deep. Assume COMP(1), COMP(2) and PI at the high level:

$$P(a|A,B,\lambda,\xi) \overset{\text{COMP}(1)}{=} P(a|A,B,\lambda) \overset{\text{PI}_{\text{high}}}{=} P(a|A,\lambda) \overset{\text{COMP}(2)}{=} P(a|A,\lambda,\xi).$$

Fundamental Stochastic Theories

Note that COMP does not force determinism, despite the assumed completeness.

- COMP blocks the decomposition of probabilities into deeper level deterministic extreme elements. Thus if a model obeys COMP and is indeterministic (the probabilities are not solely 0 and 1) then it is a fundamentally stochastic model.

$\Longrightarrow$ This formalises the notion of ‘a *complete* hidden-variable model’ in a probabilistic framework.

Motivating OI

OI is implied by COMP.

Proof: Consider the standard law of conditional probability:

$$P(a, b|A, B, \lambda) = P(a|A, B, b, \lambda)P(b|A, B, \lambda). \quad (13)$$

Consider the corollary of COMP: $P(a|A, B, \lambda, \xi) = P(a|A, B, \lambda)$.

Since ξ is some general unrestricted set of variables (other than the settings A, B and hidden variable λ) we can put the outcome b in the specification ξ. Then assuming COMP we get

$$P(a, b|A, B, \lambda) = P(a|A, B, \lambda)P(b|A, B, \lambda), \quad (14)$$

which is OI. We thus obtain OI merely from assuming COMP.

Motivating OI

It is of no importance that outcome b refers to the other spacelike separated system. If it would refer to some beable of the local system (other than the local setting a and the local measurement apparatus hidden variables, if relevant) the same would follow.

This is because COMP encodes that everything else (whether local or non-local) is redundant. Locality or nonlocality aspects associated to b are thus irrelevant.

(Note that one can include in the settings any apparatus hidden variables one may think to be relevant.)

Motivating OI

Bell remarks:

" It is notable that in this argument nothing is said about the locality, or even localizability, of the variable[s] λ. These variables could well include, for example, the quantum mechanical state vectors, which have no particular localization in ordinary space-time. It is assumed only that the outputs a and b, and the particular inputs A and B, are well localized."
(Bertlmann's socks, p. 153.)

Likewise, nothing needs to be said about the locality, or even localizability, of the variable ξ that features in the condition COMP.

Discussion

Why then such a frequent appeal to locality in motivating OI?

Two different forms of completeness/sufficiency are in play.

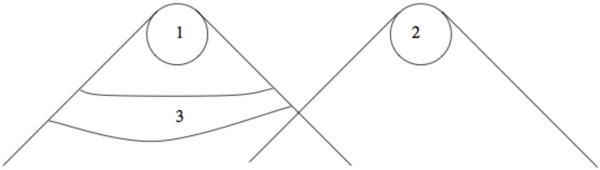
(I): A theory is complete: no other theory could even in principle provide a more accurate description.

(II): The specification of the hidden variables is complete with respect to a certain candidate theory.

- Ad I): Already dealt with, but one remark: in principle impossible to decide if one has reached the fundamental level.
- Ad II): The candidate theory may impose restictions on which hidden variables, and under what circumstances, are to be included in the complete specification.

For example: space-time restrictions. Indeed this is what many do, let us look at Bell.

On an appeal to locality



Full specification of what happens in 3 makes events in 2 irrelevant for predictions in 1 in a locally causal theory. [Figure and caption from Bell, La nouvelle Cuisine, p.106.]

“A theory is said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by a specification of values of local beables in a space-like separated region 2 when what happens in the backward light cone is already sufficiently specified, for example by a full specification of local beables in a [restricted, MPS] spacetime region 3. [. . .]

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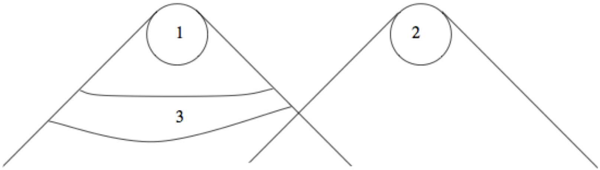

"[. . .] It is important that region 3 completely shields off from 1 the overlap of the backward light cones of 1 and 2. And it is important that events 3 be **specified completely**. Otherwise the traces in region 2 of causes of events in 1 could well supplement whatever else was being used for calculating probabilities about 1. **The hypothesis is that any such information about 2 becomes redundant when 3 is specified completely**." *[Bell, La Nouvelle cuisine, p.106. Emphasis added.]*

"Invoking local causality and **the assumed completeness of** c **and** λ, . . . we have $P(a, b|A, B, \lambda, c) = P(a|A, \lambda, c)P(b|B, \lambda, c)$"
[Ibid, p.109, emphasis added]

Consequences: Locality needed to motivate OI

A space-time structure is posited, and one for example makes assumptions about the fact that causal influences do not propagate in a spacelike fashion. One singles out a region in spacetime that should screen off. A form of locality is thus assumed. And then it should not come as a surprise that an appeal to locality is involved to get OI.

If one puts restrictions on the space-time extension of the hidden variable λ (and ξ), and if COMP **therefore** does not obtain, then it seems that an appeal to locality should be necessary to motivate OI.

Such a theory is complete(II) but incomplete(I): $\neg$ COMP.

Consequences: Locality needed to motivate OI

$$\text{OI}: \quad P(a|b, A, B, \lambda) = P(a|A, B, \lambda), \tag{15}$$

If OI is violated, how to interpret this?

$\neg$ COMP or non-local causation?

Some claim violations of OI are due to non-local causation. An other option is to say that this is only because (non-local) relevant features are left out of what is in fact the COMPlete description.

So only in case one puts a restriction (that is probably argued for by an appeal to relativity theory) on the space-time extension of the hidden variable λ, and because of that leaves out relevant hidden variables, one could be forced to make further assumptions about the prohibition of spacelike propagation, etc., in order to obtain OI. But this is a consequence of the initial restriction, i.e., the restriction to a form of (relativistic) locality forces one to appeal to it later again.

It might be that such a restriction on the space-time extension of the hidden variable λ is probably well-justified in the light of relativity theory. But even then, we have seen that in case COMP obtains, this already implies OI independent of any locality considerations.

Critical remarks

We see that an appeal to locality is made to justify OI in the case where COMP does not hold. However, Elby, Brown and Foster think it is not even reasonable to consider OI in case COMP does not hold. They remark:

” [OI] is intuitively compelling only when applied to statistically complete theories. [. . .] In brief, failure of [OI] in a statistically incomplete theory poses no challenge to our physical intuitions. The failure may indicate nothing more than our ability to reveal missing information about electron 2 by measuring a correlated system, electron 1. ” *[Elby, Brown and Foster. What makes a theory physically “complete”?, p. 979.]*

Critical remarks

Elby, Brown and Foster talk about ‘statistical completeness’. They mean the following:

“ Intuitively, a theory is [statistically] complete if no other theory could even in principle provide a more accurate and fine-grained description of nature. ” *[Ibid, p.972]*

Here we take it that COMP captures Elby-Brown-Foster’s notion of statistical completeness.

Thus in a statistically complete theory it is the case that COMP obtains and thus that OI follows. Locality is not needed!

And, conversely, following Elby-Brown- Foster it is not reasonable to consider OI in case statistical completeness [i.e., COMP] does not hold.

Conclusion and Discussion

- Two strategies have been employed:

i) An investigation of how inferences that hold on the surface level relate to those those that hold on the subsurface level.

ii) Investigate the consequences of the possibility and relevance of extra hidden variables at a deeper level.

Conclusion and Discussion

- Two strategies have been employed:

i) An investigation of how inferences that hold on the surface level relate to those those that hold on the subsurface level.

ii) Investigate the consequences of the possibility and relevance of extra hidden variables at a deeper level.

- One should not just consider the structural form of the probabilistic conditions, but also what is said about the completeness of the hidden variable specification at a certain level.

- The condition COMP captures this aspect.
  Whether or not it holds should generally be addressed in any hidden variable program.

Conclusion and Discussion

- When COMP is put in place (as it should be, we argued) we see that OI indeed is a sufficiency or completeness condition and its justification thus needs no appeal to locality or causality. One can even argue that if COMP does not hold OI is not intuitively compelling from the start.

- ▶ Open question: What happens if we bring in quantum theory? What are the repercussions of the requirement that one should also reproduce QM?

Only reproducing perfect singlet correlations was considered, and only for no-signaling correlations.

(Valentini's result: any deterministic HV theory that gives QM for some equilibrium distribution (of the HV's) must be signaling for some non-equilibrium distribution).